A New Value of the von Kármán Constant: Implications and Implementation

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ABSTRACT

The von Kármán constant k occurs throughout the mathematics that describe the atmospheric boundary layer. In particular, because k was originally included in the definition of the Obukhov length, its value has both explicit and implicit effects on the functions of Monin–Obukhov similarity theory. Although credible experimental evidence has appeared sporadically that the von Kármán constant is different than the canonical value of 0.40, the mathematics of boundary layer meteorology still retain $k = 0.40$—probably because the task of revising all of this math to implement a new value of k is so daunting. This study therefore outlines how to make these revisions in the nondimensional flux–gradient relations; in variance, covariance, and dissipation functions; and in structure parameters of Monin–Obukhov similarity theory. It also demonstrates how measured values of the drag coefficient (C_D), the transfer coefficients for sensible (C_H) and latent (C_E) heat, and the roughness lengths for wind speed (z_0), temperature (z_T), and humidity (z_Q) must be modified for a new value of the von Kármán constant. For the range of credible experimental values for k, 0.35–0.436, revised values of C_D, C_H, C_E, z_0, z_T, and z_Q could be quite different from values obtained assuming $k = 0.40$, especially if the original measurements were made in stable stratification. However, for the value of k recommended here, 0.39, no revisions to the transfer coefficients and roughness lengths should be necessary. Henceforth, use the original measured values of transfer coefficients and roughness lengths but do use similarity functions modified to reflect $k = 0.39$.

1. Introduction

The von Kármán constant, k, appears almost everywhere in the equations used in boundary layer meteorology. For example, the flux-gradient relations for the atmospheric surface layer include k as the proportionality constant that relates the wind speed profile to the surface stress and the temperature profile to the sensible heat flux. In turn, because these flux–gradient relations lead to definitions of the roughness lengths for wind speed and scalars and to expressions for momentum and heat transfer coefficients, the value of the von Kármán constant affects the reported values of these quantities.

In fact, every "universal" function in Monin–Obukhov similarity theory depends on the von Kármán constant because boundary layer meteorology evolved with k embedded in the definition of the Obukhov length (e.g., Monin and Yaglom 1971, p. 427; Businger and Yaglom 1971; Obukhov 1971; Yaglom 1977):

$$L = -\frac{\overline{\Theta_v}}{kg}\frac{u_*^3}{\overline{w\theta_v}}. \tag{1.1}$$

Here, g is the acceleration of gravity; $\overline{\Theta_v}$ is the average virtual temperature (K), u_* is the friction velocity, w is the turbulent fluctuation in vertical velocity, and θ_v is the turbulent fluctuation in virtual temperature. Because the overbar indicates averaging, $\overline{w\theta_v}$ is the vertical flux of virtual temperature.

The stratification parameter

$$\zeta = z/L, \tag{1.2}$$

where z is the measurement height, is a key variable in Monin–Obukhov similarity theory and appears in most of the similarity functions, multiplied by an empirical coefficient. Just as one example, consider the function ϕ_m, the nondimensional vertical gradient in wind speed, defined as

$$\phi_m(\zeta) = \frac{kz}{u_*}\frac{\partial U}{\partial z}, \tag{1.3}$$

where $\partial U/\partial z$ is the gradient in average wind speed in the atmospheric surface layer. In unstable stratification (i.e., $\zeta < 0$), a typical form for $\phi_m(\zeta)$ is (Paulson 1970)

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$$\phi_m(\zeta) = (1 - 16\zeta)^{-1/4}. \qquad (1.4)$$

Because k is hidden in ζ, the empirical coefficient 16 depends on the value used for k.

Although most boundary layer models and analyses use the canonical value for k "pegged at 0.4 since the Depression" (Tennekes 1973), credible alternative experimental values have been reported in the last 40 yr. Andreas et al. (2006) recently reviewed many of these modern experimental and theoretical estimates of k. To just briefly bracket the bounds of my discussion, I mention a few experimental values here.

The Kansas experiment (Haugen et al. 1971; Kaimal and Wyngaard 1990) opened the modern discussion of what the value of the von Kármán constant is when Businger et al. (1971) reported $k = 0.35$ (see also Kaimal et al. 1972; Tennekes 1973; Wyngaard et al. 1982). Oncley et al. (1996) also obtained experimental values in this range, 0.365 ± 0.015. At the other extreme, Zagarola and Smits (1998) reported one of the largest modern values of the von Kármán constant ever published, 0.436 ± 0.002, based on highly regarded measurements at high Reynolds number in the Princeton superpipe. Although McKeon et al. (2004) later reanalyzed the Zagarola and Smits data and also collected new data in the superpipe, their combined result, $k = 0.421 \pm 0.002$, is still significantly larger than the canonical value of 0.40.

Meanwhile, quite a few measurements in the atmospheric surface layer have suggested that k is only slightly smaller than 0.40. For instance, Dyer and Bradley (1982) reported that the von Kármán constant is 0.385 ± 0.021; Kondo and Sato (1982), 0.39; and Frenzen and Vogel (1995b), 0.387 ± 0.010. Finally, Andreas et al. (2006) reported on two large surface layer datasets collected over polar sea ice; their result was $k = 0.387 \pm 0.003$. Comparable work in wind tunnels likewise suggests that k is smaller than 0.40: Österlund et al. (2000) and Nagib et al. (2004) obtained 0.38, and Zanoun et al. (2003) found 0.37.

Despite these fairly compelling reports that k is not the canonical 0.40 (or 0.41; e.g., Dyer 1974), few models or boundary layer parameterizations have implemented a value other than 0.40. I can think of several reasons why. First, and most obvious, the fluid dynamics community has not come to a consensus on the true value of the von Kármán constant. Second, models and algorithms develop inertia; making even small and justifiable changes in them requires serious thought as to the consequences because the models have been reliable and tested in their existing forms. Third, since many of the existing parameterizations are empirical, it is not obvious how changing the von Kármán constant affects these parameterizations.

This paper focuses on this last impediment. If we ever decide as a community, or even as individuals, to implement a new value for the von Kármán constant, how can we do that consistently without undertaking costly new experiments and data analyses to define empirical functions that incorporate the new k value? Yaglom (1977) and Högström (1988) previously suggested ways to modify existing empirical functions for a new value of the von Kármán constant. My recommended techniques generally differ from both of theirs. I will describe Yaglom's method shortly. Högström based his modifications on the idea that bias errors in the fundamental data explained reported values of k that were different from his preferred value, 0.40, and therefore corrected empirical functions under this assumption. On the other hand, I always assume that the fundamental data are correct.

As a consequence of this study, I also consider the implications of a new value for the von Kármán constant. In essence, does using a value other than 0.40 really make any difference? For some applications the answer is yes, and for others, no.

What follows elaborates on these themes by showing how changing the von Kármán constant affects the flux–gradient relations of Monin–Obukhov similarity theory and related functions such as the nondimensional dissipation rates of turbulent kinetic energy and scalar variance and the nondimensional variance functions and structure parameters. Since the flux–gradient relations change with the von Kármán constant, previously reported values of the turbulent transfer coefficients and the roughness lengths for wind speed and scalars must also be reevaluated to reflect the new value of k. Turbulent fluxes found using the dissipation method may also need to be recomputed to be consistent with the new value of k.

I have probably overlooked other important occurrences of k, but the examples and the conversion techniques presented here should provide the basis for adapting other quantities and functions to a new value of the von Kármán constant.

2. Flux–gradient relations

Equation (1.3) is the flux–gradient relation for the wind speed profile in the atmospheric surface layer. The related flux–gradient relation for the average potential temperature (Θ) profile is

$$\phi_h(\zeta) = \frac{kz}{\theta_*}\frac{\partial \Theta}{\partial z}, \qquad (2.1)$$

where $\partial\Theta/\partial z$ is the vertical, near-surface gradient in potential temperature, $\theta_* = -\overline{w\theta}/u_*$ is a temperature flux scale, θ is the turbulent fluctuation in temperature, and ϕ_h is an empirical similarity function. Sometimes, the turbulent Prandtl number at neutral stratification, $\mathrm{Pr_{tN}}$, is included in the denominator on the right-hand side of (2.1). But because no experiments have confirmed that $\mathrm{Pr_{tN}}$ is not equal to 1 (e.g., Garratt 1992, p. 289; Grachev et al. 2007b), I leave it out of (2.1).

My assumption throughout this paper will be that the data on which empirical functions and coefficients are based are accurate; only the value of the von Kármán constant needs changing. With (1.3) and (2.1) as examples, this assumption means simply that z, u_*, θ_*, $\partial U/\partial z$, and $\partial\Theta/\partial z$ are accurate within the bounds of experimental uncertainty.

The stability parameter ζ also appears in (1.3) and (2.1), and this also derives from additional measurements of $\overline{\Theta_V}$ and $\overline{w\theta_v}$. Notice, too, that a quantity related to the stratification,

$$\hat{\zeta} = -\frac{zg}{\overline{\Theta_v}} \frac{\overline{w\theta_v}}{u_*^3}, \tag{2.2}$$

is not affected by a change in the von Kármán constant. In fact, if boundary layer meteorology had evolved with (2.2) as the stratification parameter instead of ζ, many of the conversions developed in this paper would have been unnecessary.

Because k is included in ζ, I define

$$\zeta_{\mathrm{old}} = k_{\mathrm{old}}\hat{\zeta} \quad \text{and} \tag{2.3a}$$

$$\zeta_{\mathrm{new}} = k_{\mathrm{new}}\hat{\zeta}. \tag{2.3b}$$

Here, ζ_{old} is the stratification parameter characterizing the original analysis of the flux–gradient relations: that analysis assumed the "old" value of the von Kármán constant (k_{old}; i.e., 0.40). To evaluate new ϕ_m and ϕ_h functions, I assume the original data that went into $\hat{\zeta}$ are unchanged but introducing a new von Kármán constant (k_{new}) still changes the relevant stratification (ζ_{new}).

With this terminology, (1.3) and (2.1) now become the paired equations

$$\phi_{m,\mathrm{old}}(\zeta_{\mathrm{old}}) = \frac{k_{\mathrm{old}}z}{u_*}\frac{\partial U}{\partial z}, \tag{2.4a}$$

$$\phi_{m,\mathrm{new}}(\zeta_{\mathrm{new}}) = \frac{k_{\mathrm{new}}z}{u_*}\frac{\partial U}{\partial z}, \tag{2.4b}$$

$$\phi_{h,\mathrm{old}}(\zeta_{\mathrm{old}}) = \frac{k_{\mathrm{old}}z}{\theta_*}\frac{\partial \Theta}{\partial z}, \tag{2.5a}$$

$$\phi_{h,\mathrm{new}}(\zeta_{\mathrm{new}}) = \frac{k_{\mathrm{new}}z}{\theta_*}\frac{\partial \Theta}{\partial z}. \tag{2.5b}$$

Here, (2.4a) and (2.5a) are plausible because they were based on data; but they are inaccurate because they assume the old value of the von Kármán constant. The first task is to use (2.4a) and (2.5a) to obtain (2.4b) and (2.5b), respectively, which reflect the proper value of the von Kármán constant. In (2.4) and (2.5), the "old" and "new" subscripts on the ϕ functions imply that these functions may change when we convert from k_{old} to k_{new}.

Because the data used to develop (2.4a) and (2.5a) are assumed invariant, one method for deriving $\phi_{m,\mathrm{new}}(\zeta_{\mathrm{new}})$ is to simply use this assumption to write

$$\phi_{m,\mathrm{new}}(\zeta_{\mathrm{new}}) = \frac{k_{\mathrm{new}}}{k_{\mathrm{old}}}\phi_{m,\mathrm{old}}(\zeta_{\mathrm{old}}). \tag{2.6}$$

Although this is the conversion method that Yaglom (1977) used, (2.6) is not a satisfactory result because $\phi_{m,\mathrm{new}}(\zeta_{\mathrm{new}})$ does not equal 1 for neutral stratification (where $\zeta_{\mathrm{new}} = \zeta_{\mathrm{old}} = 0$) because $k_{\mathrm{new}} \neq k_{\mathrm{old}}$. Remember, the definition of k requires that $\phi_m = 1$ at neutral stratification (e.g., Businger et al. 1971); therefore, $\phi_{m,\mathrm{old}}(\zeta_{\mathrm{old}} = 0) = 1$ is a given constraint.

Another approach is to require that the derivatives of the old and new ϕ_m functions with respect to $\hat{\zeta}$ match:

$$\frac{d\phi_{m,\mathrm{old}}(\zeta_{\mathrm{old}})}{d\hat{\zeta}} = \frac{d\phi_{m,\mathrm{new}}(\zeta_{\mathrm{new}})}{d\hat{\zeta}}. \tag{2.7}$$

Since $\hat{\zeta}$ does not depend on the von Kármán constant, this approach is what Monin–Obukhov similarity would look like if the stratification parameter had evolved without the von Kármán constant.

To proceed, I must assume a form for $\phi_{m,\mathrm{old}}$. Without much loss in generality, for unstable stratification, I use (1.4):

$$\begin{aligned}\phi_{m,\mathrm{old}}(\zeta_{\mathrm{old}}) &= (1 - \gamma_{u,\mathrm{old}}\zeta_{\mathrm{old}})^{-1/4} \\ &= (1 - \gamma_{u,\mathrm{old}}k_{\mathrm{old}}\hat{\zeta})^{-1/4}.\end{aligned} \tag{2.8}$$

Then

$$\frac{d\phi_{m,\mathrm{old}}}{d\hat{\zeta}} = \frac{\gamma_{u,\mathrm{old}}k_{\mathrm{old}}}{4}(1 - \gamma_{u,\mathrm{old}}k_{\mathrm{old}}\hat{\zeta})^{-5/4}. \tag{2.9}$$

On invoking (2.7) in terms of differentials and using the definitions (2.3), we can obtain from (2.9)

$$d\phi_{m,\mathrm{new}}(\zeta_{\mathrm{new}}) = \frac{\gamma_{u,\mathrm{old}}k_{\mathrm{old}}}{4\,k_{\mathrm{new}}}\left(1 - \frac{\gamma_{u,\mathrm{old}}k_{\mathrm{old}}}{k_{\mathrm{new}}}\zeta_{\mathrm{new}}\right)^{-5/4} d\zeta_{\mathrm{new}}. \tag{2.10}$$

Define

$$\gamma_{u,\mathrm{new}} = \frac{\gamma_{u,\mathrm{old}}k_{\mathrm{old}}}{k_{\mathrm{new}}}. \tag{2.11}$$

If we integrate (2.10) from $\zeta_{\mathrm{new}} = 0$, where $\phi_{m,\mathrm{new}}$ must equal 1, to $\zeta_{\mathrm{new}} < 0$, we get

$$\int_1^{\phi_{m,\mathrm{new}}(\zeta_{\mathrm{new}})} d\phi_{m,\mathrm{new}} = \frac{\gamma_{u,\mathrm{new}}}{4}\int_0^{\zeta_{\mathrm{new}}} (1-\gamma_{u,\mathrm{new}}\,\xi)^{-5/4}\, d\xi, \tag{2.12}$$

where ξ is the variable of integration. The result is

$$\phi_{m,\mathrm{new}}(\zeta_{\mathrm{new}}) = (1-\gamma_{u,\mathrm{new}}\zeta_{\mathrm{new}})^{-1/4}. \tag{2.13}$$

That is, the revised function $\phi_{m,\mathrm{new}}$ has the same form as the original function $\phi_{m,\mathrm{old}}$, but the coefficient multiplying the stratification parameter must be changed to (2.11).

For the numeric and graphic examples shown in the rest of this paper, I always use $k_{\mathrm{old}} = 0.40$. To bracket the range of changes resulting from a new value of the von Kármán constant, I use as the smallest reasonable value $k_{\mathrm{new}} = 0.35$, the Kansas result (Businger et al. 1971), and the largest reasonable value $k_{\mathrm{new}} = 0.436$, from Zagarola and Smits (1998). I also evaluate changes for $k_{\mathrm{new}} = 0.39$, the consensus of several surface layer experiments. For k_{new} values of 0.39, 0.35, and 0.436, $\gamma_{u,\mathrm{new}}$ is 16.4, 18.3, and 14.7, respectively, for Paulson's (1970) value $\gamma_{u,\mathrm{old}} = 16$.

The same procedure yields $\phi_{h,\mathrm{new}}(\zeta_{\mathrm{new}})$. As with (2.7), the basic constraint is that the derivatives of $\phi_{h,\mathrm{old}}$ and $\phi_{h,\mathrm{new}}$ with respect to $\hat{\zeta}$ are the same:

$$\frac{d\phi_{h,\mathrm{old}}(\zeta_{\mathrm{old}})}{d\hat{\zeta}} = \frac{d\phi_{h,\mathrm{new}}(\zeta_{\mathrm{new}})}{d\hat{\zeta}}. \tag{2.14}$$

To proceed, I use Paulson's (1970) function for $\phi_{h,\mathrm{old}}$,

$$\begin{aligned}\phi_{h,\mathrm{old}}(\zeta_{\mathrm{old}}) &= (1-\beta_{u,\mathrm{old}}\zeta_{\mathrm{old}})^{-1/2}\\ &= (1-\beta_{u,\mathrm{old}}k_{\mathrm{old}}\,\hat{\zeta})^{-1/2},\end{aligned} \tag{2.15}$$

with $\beta_{u,\mathrm{old}} = 16$. Hence, as with the sequence (2.9), (2.10), and (2.12), (2.15) leads to

$$\phi_{h,\mathrm{new}}(\zeta_{\mathrm{new}}) = (1-\beta_{u,\mathrm{new}}\zeta_{\mathrm{new}})^{-1/2}, \tag{2.16}$$

where

$$\beta_{u,\mathrm{new}} = \frac{\beta_{u,\mathrm{old}}k_{\mathrm{old}}}{k_{\mathrm{new}}}. \tag{2.17}$$

That is, $\phi_{h,\mathrm{old}}$ and $\phi_{h,\mathrm{new}}$ also have the same forms. Only the coefficient multiplying the stratification parameter changes, and the new value is the same as $\gamma_{u,\mathrm{new}}$.

Next, let us consider how the ϕ functions that describe stable stratification must be modified to implement a new value of the von Kármán constant. Among the many functions that have been formulated for ϕ_m and ϕ_h in stable stratification, I prefer Holtslag and De Bruin's (1988) result because it has desirable properties in very stable stratification (Launiainen and Vihma 1990; Jordan et al. 1999; Andreas 2002). This function is

$$\phi_{m,\mathrm{old}}(\zeta_{\mathrm{old}}) = \phi_{h,\mathrm{old}}(\zeta_{\mathrm{old}}) = 1 + \alpha_{s,\mathrm{old}}\zeta_{\mathrm{old}} + \beta_{s,\mathrm{old}}\zeta_{\mathrm{old}}(\gamma_{s,\mathrm{old}} - \delta_{s,\mathrm{old}}\zeta_{\mathrm{old}})\exp(-\delta_{s,\mathrm{old}}\zeta_{\mathrm{old}}), \tag{2.18}$$

where $\alpha_{s,\mathrm{old}} = 0.7$, $\beta_{s,\mathrm{old}} = 0.75$, $\gamma_{s,\mathrm{old}} = 6$, and $\delta_{s,\mathrm{old}} = 0.35$. For $0 \le \zeta_{\mathrm{old}} < 0.3$, (2.18) is close to the traditional Webb (1970; also Dyer 1974) function (e.g., Andreas 1998),

$$\phi_{m,\mathrm{old}}(\zeta_{\mathrm{old}}) = \phi_{h,\mathrm{old}}(\zeta_{\mathrm{old}}) = 1 + 5\zeta_{\mathrm{old}}, \tag{2.19}$$

but has more realistic values for larger ζ_{old}.

With ϕ_m as an example, I again invoke the equality of derivatives with respect to $\hat{\zeta}$, (2.7). The appropriate derivative of (2.18) is

$$\frac{d\phi_{m,\mathrm{old}}}{d\hat{\zeta}} = \alpha_{s,\mathrm{old}}k_{\mathrm{old}} + \beta_{s,\mathrm{old}}k_{\mathrm{old}}[\gamma_{s,\mathrm{old}} - 2\,\delta_{s,\mathrm{old}}k_{\mathrm{old}}\hat{\zeta} - \delta_{s,\mathrm{old}}k_{\mathrm{old}}\hat{\zeta}(\gamma_{s,\mathrm{old}} - \delta_{s,\mathrm{old}}k_{\mathrm{old}}\hat{\zeta})]\exp(-\delta_{s,\mathrm{old}}k_{\mathrm{old}}\,\hat{\zeta}). \tag{2.20}$$

Therefore,

$$d\phi_{m,\mathrm{new}}(\zeta_{\mathrm{new}}) = \{\alpha_{s,\mathrm{new}} + \beta_{s,\mathrm{new}}[\gamma_{s,\mathrm{new}} - 2\delta_{s,\mathrm{new}}\zeta_{\mathrm{new}} - \delta_{s,\mathrm{new}}\zeta_{\mathrm{new}}(\gamma_{s,\mathrm{new}} - \delta_{s,\mathrm{new}}\zeta_{\mathrm{new}})]\exp(-\delta_{s,\mathrm{new}}\zeta_{\mathrm{new}})\}\,d\zeta_{\mathrm{new}}, \tag{2.21}$$

where

$$\alpha_{s,\mathrm{new}} = \frac{\alpha_{s,\mathrm{old}} k_{\mathrm{old}}}{k_{\mathrm{new}}}, \tag{2.22a}$$

$$\beta_{s,\mathrm{new}} = \frac{\beta_{s,\mathrm{old}} k_{\mathrm{old}}}{k_{\mathrm{new}}}, \tag{2.22b}$$

$$\gamma_{s,\mathrm{new}} = \gamma_{s,\mathrm{old}}, \quad \text{and} \tag{2.22c}$$

$$\delta_{s,\mathrm{new}} = \frac{\delta_{s,\mathrm{old}} k_{\mathrm{old}}}{k_{\mathrm{new}}}. \tag{2.22d}$$

Integrating (2.21) from $\zeta_{\mathrm{new}} = 0$, where $\phi_{m,\mathrm{new}} = 1$, to $\zeta_{\mathrm{new}} > 0$ yields

$$\phi_{m,\mathrm{new}}(\zeta_{\mathrm{new}}) = 1 + \alpha_{s,\mathrm{new}}\zeta_{\mathrm{new}} + \beta_{s,\mathrm{new}}\zeta_{\mathrm{new}} \times (\gamma_{s,\mathrm{old}} - \delta_{s,\mathrm{new}}\zeta_{\mathrm{new}}) \exp(-\delta_{s,\mathrm{new}}\zeta_{\mathrm{new}}). \tag{2.23}$$

Equation (2.23) also represents $\phi_{h,\mathrm{new}}(\zeta_{\mathrm{new}})$.

As we found with unstable stratification, the new ϕ functions for stable stratification have the same functional form as the old ϕ functions. Only some of the coefficients change.

Figures 1 and 2 show old and new values of ϕ_m and ϕ_h, respectively. Both $\phi_{m,\mathrm{old}}(\zeta_{\mathrm{old}})$ and $\phi_{h,\mathrm{old}}(\zeta_{\mathrm{old}})$ are based on $k_{\mathrm{old}} = 0.40$. The $\phi_{m,\mathrm{new}}(\zeta_{\mathrm{new}})$ and $\phi_{h,\mathrm{new}}(\zeta_{\mathrm{new}})$ functions in these plots assume $k_{\mathrm{old}} = 0.40$ and treat k_{new} values of 0.39, 0.35, and 0.436. The vertical axis in each plot is logarithmic to show the relative, as opposed to the absolute, difference between old and new functions.

In Figs. 1 and 2, the effect of a new von Kármán constant is to shift the new ϕ_m and ϕ_h functions to larger or smaller absolute values of ζ than for the original functions depending on whether k_{new} is, respectively, larger or smaller than k_{old}. That shift is small, however, and is well within the scatter of data used to evaluate ϕ_m and ϕ_h (e.g., Högström 1996; Grachev et al. 2007a). Consequently, although the proper value of k ensures that both ϕ_m and ϕ_h equal 1 for neutral stratification, changing the value of k will have an insignificant effect on the difference between stability dependencies between old and new values of ϕ_m and ϕ_h.

A result of the analysis in this section that will be important later is

$$\phi_{m,\mathrm{new}}(\zeta_{\mathrm{new}}) = \phi_{m,\mathrm{old}}(\zeta_{\mathrm{old}}) \quad \text{and} \tag{2.24a}$$

$$\phi_{h,\mathrm{new}}(\zeta_{\mathrm{new}}) = \phi_{h,\mathrm{old}}(\zeta_{\mathrm{old}}), \tag{2.24b}$$

for both stable and unstable stratification. As an example of how to establish this, consider (2.13). It gives

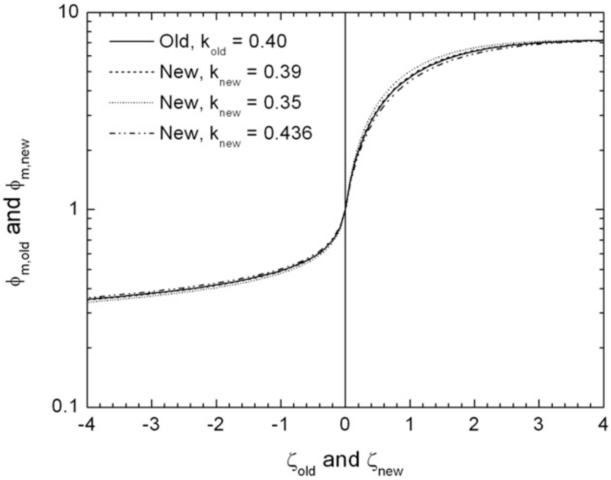

FIG. 1. Comparison of old and new values of ϕ_m. The ''old'' function is (2.8) with $\gamma_{u,\mathrm{old}} = 16$ for unstable stratification and is (2.18) for stable stratification; the von Kármán constant for these is 0.40, and the stratification parameter is ζ_{old}. The ''new'' functions at stratification ζ_{new} are (2.13) and (2.23) with coefficients given by (2.11) and (2.22), respectively, and for three k_{new} values, as noted. For these new functions, k_{old} is always 0.40.

$$\phi_{m,\mathrm{new}}(\zeta_{\mathrm{new}}) = \left[1 - \left(\frac{\gamma_{u,\mathrm{old}} k_{\mathrm{old}}}{k_{\mathrm{new}}}\right)(k_{\mathrm{new}}\,\widehat{\zeta})\right]^{-1/4} = (1 - \gamma_{u,\mathrm{old}}\zeta_{\mathrm{new}})^{-1/4} = \phi_{m,\mathrm{old}}(\zeta_{\mathrm{old}}). \tag{2.25}$$

On integration, (2.4) and (2.5) yield expressions for the average profiles of wind speed and temperature. Equations (2.4) and (2.5) can also be recast into expressions for the turbulent diffusivities for momentum and heat. The appendix shows how to reformulate these profile equations and diffusivities to reflect a new value of the von Kármán constant.

3. Variance functions

The nondimensional variance and covariance are other Monin–Obukhov similarity functions that will change with a new value of the von Kármán constant. Suppose x and y are turbulent fluctuations in two variables and x_* and y_* are corresponding flux scales. The general form of the nondimensional covariance similarity function is

$$\phi_{\mathrm{xy}}(\zeta) = \frac{\overline{xy}}{x_* y_*}. \tag{3.1}$$

One common example of this function is the temperature–humidity covariance (e.g., Andreas et al. 1998):

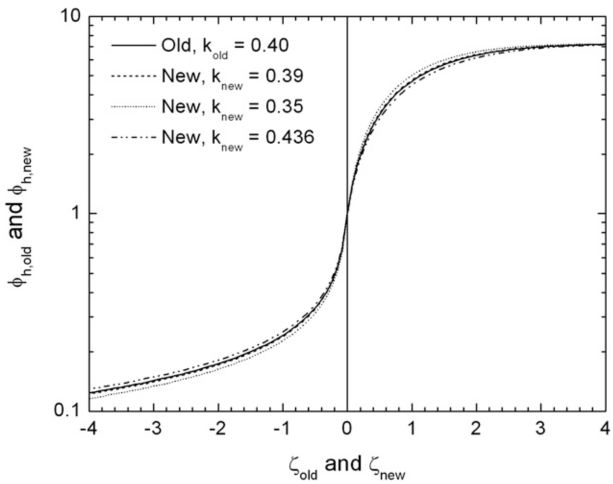

FIG. 2. As in Fig. 1, except this shows ϕ_h. In unstable stratification, $\phi_{h,\mathrm{old}}$ is (2.15) with $\beta_{u,\mathrm{old}} = 16$, and $\phi_{h,\mathrm{new}}$ is (2.16) with $\beta_{u,\mathrm{new}}$ given by (2.17).

$$\phi_{\theta\mathrm{q}}(\zeta) = \frac{\overline{\theta q}}{\theta_* q_*}. \tag{3.2}$$

Here, q is the turbulent fluctuation in specific humidity, and $q_* = -\overline{wq}/u_*$.

More commonly though, (3.1) is formulated for variance. With vertical velocity variance, $\sigma_w^2 = \overline{w^2}$, as an example, (3.1) reduces to

$$\phi_{\mathrm{ww}}(\zeta) = \frac{\sigma_w^2}{u_*^2}. \tag{3.3}$$

Because ζ includes the von Kármán constant, these variance and covariance functions must change if we change the von Kármán constant.

Sorbjan (1989, 74–76), Kaimal and Finnigan (1994, p. 16), and Andreas et al. (1998), among many others, suggest that $\phi_{\mathrm{xy,old}}$ has the general form

$$\phi_{\mathrm{xy,old}}(\zeta_{\mathrm{old}}) = a_{\mathrm{old}}(b_{\mathrm{old}} + c_{\mathrm{old}}\zeta_{\mathrm{old}}^{d_{\mathrm{old}}})^{e_{\mathrm{old}}}, \tag{3.4}$$

where a_{old}, b_{old}, c_{old}, d_{old}, and e_{old} are empirical constants that may change with x and y.

To find $\phi_{\mathrm{xy,new}}(\zeta_{\mathrm{new}})$, I invoke the same constraint that I did for finding $\phi_{m,\mathrm{new}}$ and $\phi_{h,\mathrm{new}}$. That is, the derivatives with respect to $\hat{\zeta}$ must match:

$$\frac{d\phi_{\mathrm{xy,old}}(\zeta_{\mathrm{old}})}{d\hat{\zeta}} = \frac{d\phi_{\mathrm{xy,new}}(\zeta_{\mathrm{new}})}{d\hat{\zeta}}. \tag{3.5}$$

From (3.4),

$$\frac{d\phi_{\mathrm{xy,old}}(\zeta_{\mathrm{old}})}{d\hat{\zeta}} = a_{\mathrm{old}}\, e_{\mathrm{old}}(b_{\mathrm{old}} + c_{\mathrm{old}}k_{\mathrm{old}}^{d_{\mathrm{old}}}\hat{\zeta}^{d_{\mathrm{old}}})^{e_{\mathrm{old}}-1} \times (c_{\mathrm{old}}d_{\mathrm{old}}k_{\mathrm{old}}^{d_{\mathrm{old}}}\hat{\zeta}^{d_{\mathrm{old}}-1}). \tag{3.6}$$

Combining (3.5) and (3.6) yields

$$d\phi_{\mathrm{xy,new}}(\zeta_{\mathrm{new}}) = a_{\mathrm{old}}e_{\mathrm{old}}\left(b_{\mathrm{old}} + \frac{c_{\mathrm{old}}k_{\mathrm{old}}^{d_{\mathrm{old}}}}{k_{\mathrm{new}}^{d_{\mathrm{old}}}}\,\zeta_{\mathrm{new}}^{d_{\mathrm{old}}}\right)^{e_{\mathrm{old}}-1} \times \left(\frac{c_{\mathrm{old}}d_{\mathrm{old}}k_{\mathrm{old}}^{d_{\mathrm{old}}}}{k_{\mathrm{new}}^{d_{\mathrm{old}}}}\,\zeta_{\mathrm{new}}^{d_{\mathrm{old}}-1}\right)d\zeta_{\mathrm{new}}. \tag{3.7}$$

Consequently, define

$$a_{\mathrm{new}} = a_{\mathrm{old}}, \tag{3.8a}$$

$$b_{\mathrm{new}} = b_{\mathrm{old}}, \tag{3.8b}$$

$$c_{\mathrm{new}} = c_{\mathrm{old}}\left(\frac{k_{\mathrm{old}}}{k_{\mathrm{new}}}\right)^{d_{\mathrm{old}}}, \tag{3.8c}$$

$$d_{\mathrm{new}} = d_{\mathrm{old}}, \quad \text{and} \tag{3.8d}$$

$$e_{\mathrm{new}} = e_{\mathrm{old}}. \tag{3.8e}$$

No reason exists why $\phi_{\mathrm{xy,old}}(\zeta_{\mathrm{old}} = 0)$ should not equal $\phi_{\mathrm{xy,new}}(\zeta_{\mathrm{new}} = 0)$. Therefore, integrating (3.7) from $\zeta_{\mathrm{new}} = 0$ to ζ_{new} gives

$$\int_{a_{\mathrm{new}}\,b_{\mathrm{new}}^{e_{\mathrm{new}}}}^{\phi_{\mathrm{xy,new}}(\zeta_{\mathrm{new}})} d\phi_{\mathrm{xy,new}} = a_{\mathrm{new}}c_{\mathrm{new}}d_{\mathrm{new}}e_{\mathrm{new}} \times \int_0^{\zeta_{\mathrm{new}}} (b_{\mathrm{new}} + c_{\mathrm{new}}\xi^{d_{\mathrm{new}}})^{e_{\mathrm{new}}-1}\xi^{d_{\mathrm{new}}-1}\,d\xi, \tag{3.9}$$

where ξ is again the variable of integration. Completing the integration in (3.9) yields

$$\phi_{\mathrm{xy,new}}(\zeta_{\mathrm{new}}) = a_{\mathrm{old}}(b_{\mathrm{old}} + c_{\mathrm{new}}\zeta_{\mathrm{new}}^{d_{\mathrm{old}}})^{e_{\mathrm{old}}}, \tag{3.10}$$

where only c_{new} has a different value than in (3.4). That is, the old and new ϕ_{xy} functions have the same forms, with only one different constant.

Let us look at how changing the von Kármán constant affects variance and covariance functions by considering just one example, ϕ_{ww}. Many forms exist for all the variance and covariance functions, but there is some consensus as to the form of $\phi_{\mathrm{ww,old}}(\zeta_{\mathrm{old}})$. From Panofsky and Dutton (1984, p. 161), Sorbjan (1989, 75–76), Kaimal and Finnigan (1994, p. 16), and Andreas et al. (1998), I put together these typical functions,

$$\phi_{\mathrm{ww,old}}(\zeta_{\mathrm{old}}) = 1.56(1 + 3\zeta_{\mathrm{old}})^{2/3} \quad \text{for } \zeta_{\mathrm{old}} \le 0, \tag{3.11a}$$

$$\phi_{ww,old}(\zeta_{old}) = 1.56(1 + 0.2\zeta_{old})^2 \quad \text{for } \zeta_{old} \geq 0. \tag{3.11b}$$

Using (3.4), (3.8), and (3.10) immediately converts $\phi_{ww,old}$ to

$$\phi_{ww,new}(\zeta_{new}) = 1.56\left[1 + 3\left(\frac{k_{old}}{k_{new}}\right)\zeta_{new}\right]^{2/3} \quad \text{for } \zeta_{new} \leq 0, \tag{3.12a}$$

$$\phi_{ww,new}(\zeta_{new}) = 1.56\left[1 + 0.2\left(\frac{k_{old}}{k_{new}}\right)\zeta_{new}\right]^{2} \quad \text{for } \zeta_{new} \geq 0. \tag{3.12b}$$

Figure 3 shows $\phi_{ww,old}$ and versions of $\phi_{ww,new}$ for three different values of a new von Kármán constant. If k_{new} changes only to 0.39, Fig. 3 shows that $\phi_{ww,old}$ and $\phi_{ww,new}$ are negligibly different. But for the largest and smallest values of k_{new}, $\phi_{ww,new}$ can differ by about 10% from $\phi_{ww,old}$ for very unstable and very stable stratification.

Data plots of ϕ_{ww}—for example in Wyngaard et al. (1971a), Panofsky et al. (1977), Panofsky and Dutton (1984, p. 161), Högström (1990), and Andreas et al. (1998)—tend to show fairly small scatter in unstable stratification as ζ decreases. The scatter is generally less than or approximately equal to the spread in the ϕ_{ww} functions shown on the unstable side of Fig. 3. In other words, the value of the von Kármán constant selected for an analysis of ϕ_{ww} strongly influences the value obtained for the coefficient multiplying ζ—that is, c_{new}.

On the stable side of ϕ_{ww} data plots, however, the data are usually far more scattered than the spread in the ϕ_{ww} curves in Fig. 3. Here, ϕ_{ww} is not well constrained, and the choice of k has an insignificant effect on the uncertainty in evaluations of c_{new}.

4. Dissipation functions

The nondimensional rate for the dissipation of turbulent kinetic energy ε is another Monin–Obukhov similarity function that depends on the von Kármán constant. It is defined as

$$\phi_\varepsilon(\zeta) = \frac{kz\varepsilon}{u_*^3}. \tag{4.1}$$

Sorbjan (1989, 75–76) summarizes several expressions for $\phi_{\varepsilon,old}(\zeta_{old})$; these take the general form

$$\phi_{\varepsilon,old}(\zeta_{old}) = (1 + a_{\varepsilon,old}\zeta_{old}^{b_{\varepsilon,old}})^{c_{\varepsilon,old}}. \tag{4.2}$$

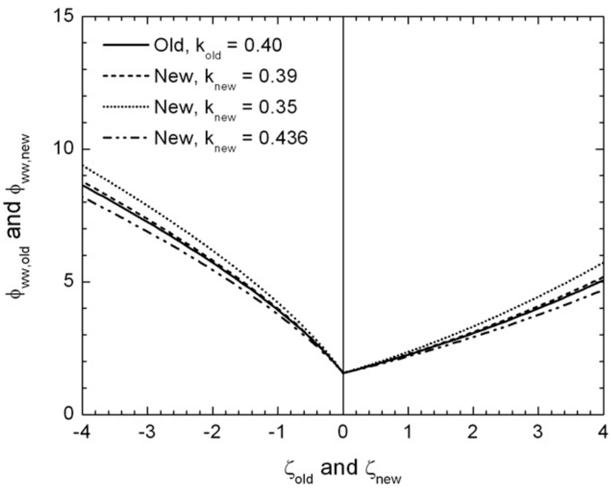

FIG. 3. Comparison of old and new functional forms for the vertical velocity variance. The "old" function is (3.11) and assumes the von Kármán constant to be 0.40. The "new" function is (3.12). For these, k_{old} is always 0.40; k_{new} values are as noted in the legend.

Because all of the functions that Sorbjan reviews equal one at neutral stratification (cf. Kaimal and Finnigan 1994, p. 16), to obtain $\phi_{\varepsilon,new}(\zeta_{new})$, I must again invoke the equality of derivatives with respect to $\widehat{\zeta}$:

$$\frac{d\phi_{\varepsilon,old}(\zeta_{old})}{d\widehat{\zeta}} = \frac{d\phi_{\varepsilon,new}(\zeta_{new})}{d\widehat{\zeta}}. \tag{4.3}$$

Inserting (4.2) into (4.3) yields

$$\frac{d\phi_{\varepsilon,old}(\zeta_{old})}{d\widehat{\zeta}} = c_{\varepsilon,old}(1 + a_{\varepsilon,old}k_{old}^{b_{\varepsilon,old}}\widehat{\zeta}^{b_{\varepsilon,old}})^{c_{\varepsilon,old}-1} \times (a_{\varepsilon,old}k_{old}^{b_{\varepsilon,old}}b_{\varepsilon,old}\widehat{\zeta}^{b_{\varepsilon,old}-1}). \tag{4.4}$$

Consequently,

$$d\phi_{\varepsilon,new}(\zeta_{new}) = c_{\varepsilon,new}(1 + a_{\varepsilon,new}\zeta_{new}^{b_{\varepsilon,new}})^{c_{\varepsilon,new}-1} \times (a_{\varepsilon,new}b_{\varepsilon,new}\zeta_{new}^{b_{\varepsilon,new}-1})d\zeta_{new}, \tag{4.5}$$

where

$$a_{\varepsilon,new} = a_{\varepsilon,old}\left(\frac{k_{old}}{k_{new}}\right)^{b_{\varepsilon,old}}, \tag{4.6a}$$

$$b_{\varepsilon,new} = b_{\varepsilon,old}, \quad \text{and} \tag{4.6b}$$

$$c_{\varepsilon,new} = c_{\varepsilon,old}. \tag{4.6c}$$

As before, integrating (4.5) from $\zeta_{new} = 0$, where $\phi_{\varepsilon,new} = 1$, to ζ_{new} gives

$$\phi_{\varepsilon,new}(\zeta_{new}) = (1 + a_{\varepsilon,new}\zeta_{new}^{b_{\varepsilon,old}})^{c_{\varepsilon,old}}. \tag{4.7}$$

That is, the new ϕ_ε function has the same form as the old function, and only the coefficient $a_{\varepsilon,\mathrm{new}}$ changes from its old value.

The nondimensional dissipation rate for scalar variance ϕ_N is a related similarity function. This is defined as

$$\phi_N(\zeta) = \frac{kzN_\theta}{u_* \theta_*^2}, \tag{4.8}$$

where (4.8) explicitly considers the dissipation rate for temperature variance, N_θ. Nondimensional dissipation rates for other scalars, such as humidity, refractive index, and the temperature–humidity covariance, have similar forms (e.g., Andreas 1988).

The production-equals-dissipation form of the scalar variance budget (e.g., Champagne et al. 1977; Andreas 1987b) implies that

$$-2\,\overline{w\theta}\,\frac{\partial \Theta}{\partial z} = N_\theta. \tag{4.9}$$

Inserting (2.1) here converts (4.8) to

$$\phi_{N,\mathrm{old}}(\zeta_{\mathrm{old}}) = 2\phi_{h,\mathrm{old}}(\zeta_{\mathrm{old}}) \quad \text{and} \tag{4.10a}$$

$$\phi_{N,\mathrm{new}}(\zeta_{\mathrm{new}}) = 2\phi_{h,\mathrm{new}}(\zeta_{\mathrm{new}}). \tag{4.10b}$$

Thus, $\phi_{N,\mathrm{new}}(\zeta_{\mathrm{new}})$ easily derives from my previous evaluations of $\phi_{h,\mathrm{new}}(\zeta_{\mathrm{new}})$.

The multiplicative 2s in (4.10) emphasize an important point here. The N_θ refers to the dissipation rate of temperature variance. Sometimes the scalar dissipation rate is defined as the dissipation rate of one-half the scalar variance in analogy with the dissipation rate for turbulent kinetic energy (e.g., Paquin and Pond 1971; Williams and Paulson 1977; Edson and Fairall 1998). With this definition for N_θ, the 2s would disappear.

Figures 4 and 5 show old and new values for ϕ_ε and ϕ_N. For $\phi_{\varepsilon,\mathrm{old}}(\zeta_{\mathrm{old}})$, I use Wyngaard and Coté's (1971) function, which is one of the functions that Sorbjan (1989, 75–76) lists and is also Kaimal and Finnigan's (1994, p. 16) recommendation for unstable stratification:

$$\phi_{\varepsilon,\mathrm{old}}(\zeta_{\mathrm{old}}) = (1 + 0.5\zeta_{\mathrm{old}}^{2/3})^{3/2} \quad \text{for } \zeta_{\mathrm{old}} \le 0, \tag{4.11a}$$

$$\phi_{\varepsilon,\mathrm{old}}(\zeta_{\mathrm{old}}) = (1 + 2.5\zeta_{\mathrm{old}}^{3/5})^{3/2} \quad \text{for } \zeta_{\mathrm{old}} \ge 0. \tag{4.11b}$$

From (4.7) and (4.6), for a new value of the von Kármán constant, these become

$$\phi_{\varepsilon,\mathrm{new}}(\zeta_{\mathrm{new}}) = \left[1 + 0.5\left(\frac{k_{\mathrm{old}}}{k_{\mathrm{new}}}\right)^{2/3} \zeta_{\mathrm{new}}^{2/3}\right]^{3/2} \quad \text{for } \zeta_{\mathrm{new}} \le 0, \tag{4.12a}$$

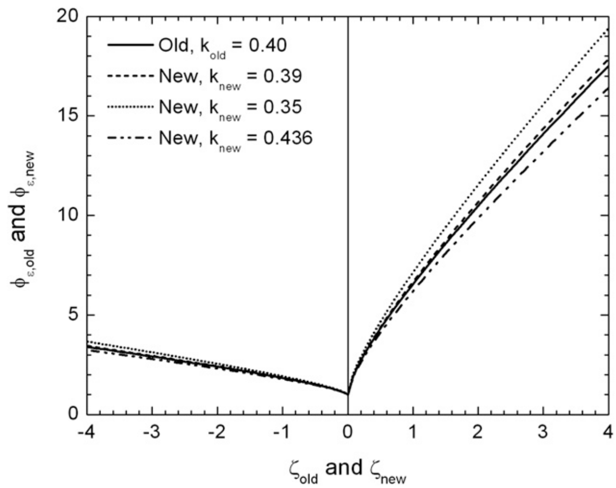

FIG. 4. Comparison of old and new values for the nondimensional dissipation rate of turbulent kinetic energy for several choices of the von Kármán constant. The functions plotted are (4.11) and (4.12).

$$\phi_{\varepsilon,\mathrm{new}}(\zeta_{\mathrm{new}}) = \left[1 + 2.5\left(\frac{k_{\mathrm{old}}}{k_{\mathrm{new}}}\right)^{3/5} \zeta_{\mathrm{new}}^{3/5}\right]^{3/2} \quad \text{for } \zeta_{\mathrm{new}} \ge 0. \tag{4.12b}$$

For $\phi_{N,\mathrm{old}}$ and $\phi_{N,\mathrm{new}}$ in (4.10), I substitute the same functions used for $\phi_{h,\mathrm{old}}$ and $\phi_{h,\mathrm{new}}$ in section 2.

The ϕ_ε functions in Fig. 4 differ little with the von Kármán constant for unstable stratification. In stable stratification, the ϕ_ε functions diverge more, but too few observations of ϕ_ε exist for $\zeta > 2$ for me to judge how the spread in the functions compares with the spread in the data. From Fig. 4 it is clear, though, that changing from $k_{\mathrm{old}} = 0.40$ to the recommended value $k_{\mathrm{new}} = 0.39$ will have an almost negligible effect on $\phi_{\varepsilon,\mathrm{new}}$.

The ϕ_N values in Fig. 5 change little with the von Kármán constant. In contrast to Fig. 4, even in stable stratification, the ϕ_N values differ little among the various values of the von Kármán constant. Because ϕ_N increases only slowly in the range $\zeta > 2$, the various functions almost converge here. Consequently, for the values considered here, the choice of von Kármán constant has little effect on typical ϕ_N functions.

5. Structure parameters

I focus here on the scalar structure parameters because these often see use in inferring surface fluxes from electro-optical propagation measurements (e.g., Wyngaard and Clifford 1978; Andreas 1989; Green et al. 1994; De Bruin et al. 2002) and through the dissipation method (e.g., Fairall and Larsen 1986; Edson and Fairall 1998).

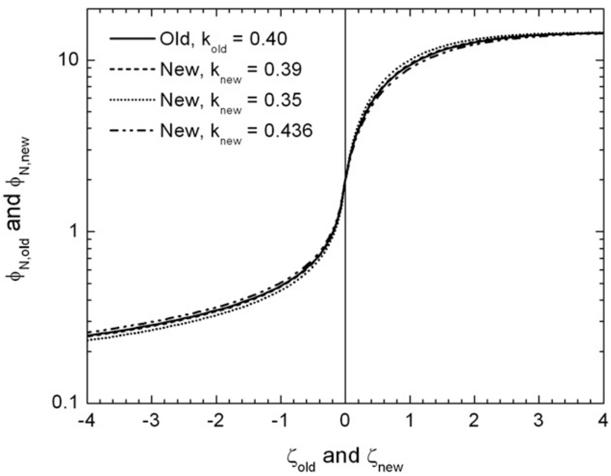

FIG. 5. Effect of changing the von Kármán constant on the nondimensional dissipation rate of scalar variance. The functions plotted are (4.10) with $\phi_{h,\mathrm{old}}(\zeta_{\mathrm{old}})$ and $\phi_{h,\mathrm{new}}(\zeta_{\mathrm{new}})$ taken as (2.15), (2.16), (2.18), and (2.23).

One easy way to derive the similarity forms for the scalar structure parameters is from the scalar spectrum. In the inertial–convective subrange, the one-dimensional temperature spectrum (Φ_θ), for example, is (Corrsin 1951)

$$\Phi_\theta(\kappa) = \beta N_\theta \varepsilon^{-1/3}\kappa^{-5/3}, \tag{5.1}$$

where κ is the turbulence wavenumber and β is called the Kolmogorov or Corrsin constant. But the same spectrum can be written in terms of the temperature structure parameter C_θ^2 (e.g., Hill and Clifford 1978; Andreas 1987b):

$$\Phi_\theta(\kappa) = 0.249 C_\theta^2 \kappa^{-5/3}. \tag{5.2}$$

Consequently,

$$C_\theta^2 = \frac{\beta N_\theta}{0.249\varepsilon^{1/3}}. \tag{5.3}$$

Similar expressions represent the structure parameters for humidity, refractive index, and temperature–humidity covariance (Andreas 1987b).

The similarity functions for ε [i.e., (4.1)] and N_θ [i.e., (4.8)] convert (5.3) to a nondimensional structure parameter (cf. Panofsky and Dutton 1984, 182–185; Andreas 1988):

$$g(\zeta) = \frac{z^{2/3}C_\theta^2}{\theta_*^2} = \frac{2\beta\phi_h(\zeta)}{0.249k^{2/3}\phi_\varepsilon^{1/3}(\zeta)}. \tag{5.4}$$

This derivation of a similarity expression for the structure parameter contrasts with derivations based strictly on plotting measurements of the nondimensional structure parameter [the middle term in (5.4)] versus ζ (see, e.g., Wyngaard et al. 1971b; Davidson et al. 1978; Fairall et al. 1980; Kohsiek 1982; and Hill and Ochs 1992). I prefer (5.4) to these empirical expressions because it explicitly shows that $g(\zeta)$ depends multiplicatively on the von Kármán constant.

Moreover, from the expressions that I have already developed, it is easy to write expressions for the old and new forms for $g(\zeta)$:

$$g_{\mathrm{old}}(\zeta_{\mathrm{old}}) \equiv \frac{2\beta\phi_{h,\mathrm{old}}(\zeta_{\mathrm{old}})}{0.249k_{\mathrm{old}}^{2/3}\phi_{\varepsilon,\mathrm{old}}^{1/3}(\zeta_{\mathrm{old}})} \quad \text{and} \tag{5.5a}$$

$$g_{\mathrm{new}}(\zeta_{\mathrm{new}}) \equiv \frac{2\beta\phi_{h,\mathrm{new}}(\zeta_{\mathrm{new}})}{0.249k_{\mathrm{new}}^{2/3}\phi_{\varepsilon,\mathrm{new}}^{1/3}(\zeta_{\mathrm{new}})}. \tag{5.5b}$$

Here $\beta = 0.40$ (Andreas 1987b). Although Eqs. (5.5) are for the temperature structure parameter, because ϕ_h and β seem to be the same for all scalars, they also apply to structure parameters for humidity, temperature–humidity covariance, and refractive index.

Figure 6 shows (5.5) for $k_{\mathrm{old}} = 0.40$ and for three values of k_{new}. On the unstable side of the plot, the four curves are nearly indistinguishable except near $\zeta = 0$. On the stable side, the curves separate more but do not spread beyond the scatter in typical data. Notice, though, that the intercept at $\zeta = 0$ does depend significantly on k. At neutral stratification, for k values of 0.40, 0.39, 0.35, and 0.436, the g functions are, respectively, 5.92, 6.02, 6.47, and 5.59.

6. Drag coefficient and roughness length z_0

Bulk turbulent flux algorithms relate the turbulent surface fluxes of momentum (τ), sensible heat (H_s), and latent heat (H_L) to mean meteorological quantities through transfer coefficients. Typical equations are

$$\tau = \rho u_*^2 = \rho C_{\mathrm{Dr}} U_r^2, \tag{6.1a}$$

$$H_s = -\rho c_p u_* \theta_* = \rho c_p C_{\mathrm{Hr}} U_r(\Theta_s - \Theta_r), \tag{6.1b}$$

$$H_L = -\rho L_v u_* q_* = \rho L_v C_{\mathrm{Er}} U_r(Q_s - Q_r). \tag{6.1c}$$

In these, ρ is the air density; c_p is the specific heat of air at constant pressure; L_v is the latent heat of vaporization (or sublimation); U_r, Θ_r, and Q_r are the average wind speed, potential temperature, and specific humidity, respectively, at reference height $z = r$; and Θ_s and Q_s are, respectively, the temperature and specific humidity at the surface.

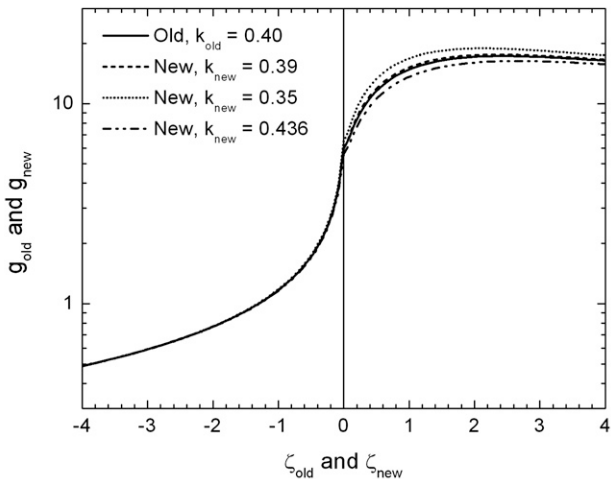

FIG. 6. Comparison of old and new values for the nondimensional structure parameter for several choices of the von Kármán constant. The plotted functions are (5.5).

Monin–Obukhov similarity theory provides expressions for the drag coefficient (C_{Dr}) and for the sensible (C_{Hr}) and latent (C_{Er}) heat transfer coefficients in (6.1) (e.g., Garratt 1992, 52–57):

$$C_{Dr} = \frac{k^2}{[\ln(r/z_0) - \psi_m(\zeta)]^2}, \qquad (6.2a)$$

$$C_{Hr} = \frac{k\,C_{Dr}^{1/2}}{[\ln(r/z_T) - \psi_h(\zeta)]}, \quad \text{and} \qquad (6.2b)$$

$$C_{Er} = \frac{k\,C_{Dr}^{1/2}}{[\ln(r/z_Q) - \psi_h(\zeta)]}. \qquad (6.2c)$$

These coefficients are appropriate only when matched with observations of U_r, Θ_r, and Q_r at height r. Here, too, ζ is defined as r/L; and z_0, z_T, and z_q are, respectively, the roughness lengths for the wind speed, temperature, and humidity profiles.

In practice, (6.1) and (6.2) must be solved iteratively because they are coupled through ζ, which includes the fluxes being estimated [see (1.1)].

Because the von Kármán constant appears in (6.2), published values of the transfer coefficients and the roughness lengths may need to be modified if we adopt a new value for k. Let us look first at how the drag coefficient and z_0 might change.

Because all of the transfer coefficients in (6.2) depend on stratification and the measurement height r, to compare coefficients collected under different conditions, we commonly report values converted to neutral stability (i.e., $\zeta = 0$) and to a standard reference height of 10 m. From (6.2a), this neutral-stability, 10-m drag coefficient is

$$C_{DN10} = \frac{k^2}{[\ln(10/z_0)]^2}, \qquad (6.3)$$

where z_0 must be in meters.

Consequently, old and new values of the drag coefficient take the forms [from (6.2a)]

$$C_{Dr}^{1/2} = \frac{1}{C_{DN10,old}^{-1/2} + k_{old}^{-1}[\ln(r/10) - \psi_{m,old}(\zeta_{old})]} \qquad (6.4a)$$

and

$$C_{Dr}^{1/2} = \frac{1}{C_{DN10,new}^{-1/2} + k_{new}^{-1}[\ln(r/10) - \psi_{m,new}(\zeta_{new})]}. \qquad (6.4b)$$

Notice here that $C_{Dr}^{1/2}$ is the same in both (6.4a) and (6.4b) because, from (6.1a), it derives strictly from data and is therefore independent of the von Kármán constant.

As a result, I can immediately equate (6.4a) and (6.4b):

$$C_{DN10,new}^{-1/2} = C_{DN10,old}^{-1/2} + \left(\frac{k_{new} - k_{old}}{k_{old}k_{new}}\right)[\ln(r/10) - \psi_{m,old}(\zeta_{old})]. \qquad (6.5)$$

In deriving (6.5), I used the equality (A.8a). In turn, from (6.5), the relation between old and new values of C_{DN10} is

$$C_{DN10,new} = \frac{1}{\left\{C_{DN10,old}^{-1/2} + \left(\frac{k_{new} - k_{old}}{k_{old}k_{new}}\right)[\ln(r/10) - \psi_{m,old}(\zeta_{old})]\right\}^2}. \qquad (6.6)$$

An unfortunate consequence of (6.6) is that revised values of the neutral-stability, 10-m drag coefficient depend on the stratification in which the original values were measured (i.e., on ζ_{old}). The revised values are also sensitive to the original measurement height r. Notice though that if the original measurements were made in neutral stratification [i.e., $\psi_{m,old}(\zeta_{old}) = 0$] and at a reference height of 10 m, the new values and the old values

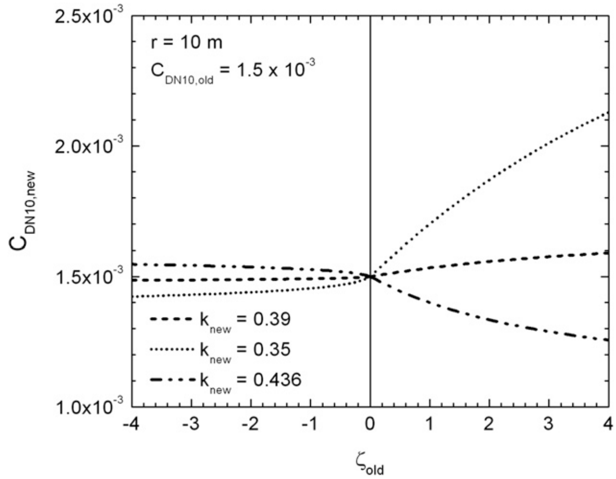

FIG. 7. Values of the neutral-stability, 10-m drag coefficient, $C_{DN10,new}$, for three new values of the von Kármán constant, computed from (6.6). The original or old value of the transfer coefficient, $C_{DN10,old}$, is assumed to be 1.5×10^{-3} and was based on data collected at 10 m and an old value for the von Kármán constant of $k_{old} = 0.40$; $C_{DN10,new}$ also depends on the stratification (ζ_{old}) in which the original data were obtained.

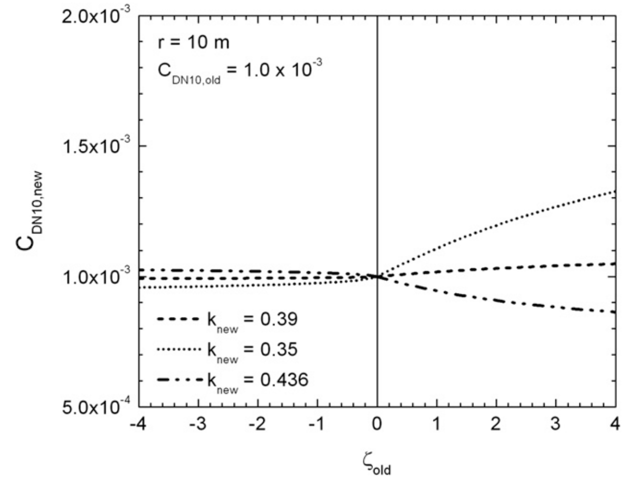

FIG. 8. As in Fig. 7, but $C_{DN10,old} = 1.0 \times 10^{-3}$.

are the same. Figures 7–9 demonstrate these and other results. The calculations presented used the functions given in (A.3a) and (A.5) for $\psi_{m,old}(\zeta_{old})$.

Figure 7 shows $C_{DN10,new}$ as a function of the original stratification, ζ_{old}, for an original measurement height of 10 m and if $C_{DN10,old}$ was evaluated to be 1.5×10^{-3}. The plot shows three possible values of k_{new}, 0.39, 0.35, and 0.436. On the unstable side of the plot, $C_{DN10,new}$ changes, at most, by 6% for the range of plausible von Kármán constants shown. Decreasing the value of the von Kármán constant from $k_{old} = 0.40$ decreases $C_{DN10,new}$; increasing the von Kármán constant increases $C_{DN10,new}$. Moreover, $C_{DN10,new}$ is not very sensitive to the original stratification if that stratification was unstable.

On the stable side of Fig. 7, on the other hand, $C_{DN10,new}$ is sensitive to both the original stratification and the new value of the von Kármán constant. Also in contrast to unstable stratification, $C_{DN10,new}$ increases if k_{new} decreases from $k_{old} = 0.40$ and decreases if k_{new} increases. The fact that $\psi_{m,old}$ in (6.6) is positive in unstable stratification and negative in stable stratification explains this behavior.

For the recommended value for k_{new}, 0.39, $C_{DN10,new}$ increases by a maximum of only 6% over $C_{DN10,old}$ for the range of ζ_{old} values shown on the stable side of Fig. 7. For larger and smaller values of k_{new}, however, $C_{DN10,new}$ can differ significantly from $C_{DN10,old}$. In essence, this strong response of $C_{DN10,new}$ to k_{new} in stable stratification is related to the fact that the vertical velocity gradient gets increasingly steeper as the stratification increases. In other words, this behavior is a consequence of $\psi_{m,old}$ in (6.6).

Figure 8 shows a similar plot of $C_{DN10,new}$ but presumes $C_{DN10,old} = 1.0 \times 10^{-3}$. This figure shows behavior like that in Fig. 7. One difference, though, is that because $C_{DN10,old}$ is smaller here, the relative changes in $C_{DN10,new}$ are not as large. For example, for $k_{new} = 0.39$, $C_{DN10,new}$ is only about 4% larger than $C_{DN10,old}$ at $\zeta_{old} = 4$, as compared with about 6% in Fig. 7.

Figure 9 is the last plot in this sequence. It shows $C_{DN10,new}$ if $C_{DN10,old}$ is again presumed to be 1.5×10^{-3} but if the original measurement height was 20 m. In this figure, $C_{DN10,new}$ does not equal $C_{DN10,old}$ at neutral stratification, as in Figs. 7 and 8. On the unstable side of Fig. 9, $C_{DN10,new}$ does not differ as much from $C_{DN10,old}$ as it did in Fig. 7, which also presumes $C_{DN10,old} = 1.5 \times 10^{-3}$. But on the stable side, $C_{DN10,new}$ differs even more from $C_{DN10,old}$.

In summary, changing to a new value of the von Kármán constant would probably not require any change in the value of C_{DN10} used in a bulk flux algorithm if the original data were collected in unstable stratification. The scatter in measurements of C_{DN10} is usually much more than the spread in the curves for unstable stratification in Figs. 7–9.

For original data collected in stable stratification, on the other hand, the required modification in $C_{DN10,new}$ could be quite large. But for a small change in the von Kármán constant—say, from $k_{old} = 0.40$ to $k_{new} = 0.39$—the modification to $C_{DN10,new}$ might be again negligible because typical scatter in the data far exceeds the deviation of $C_{DN10,new}$ from $C_{DN10,old}$ for $k_{new} = 0.39$ in Figs. 7–9.

Although Figs. 7–9 are not directly adaptable, they suggest possible biases in reported values of C_{DN10}. If $C_{DN10,old}$ values were based on a value of the von Kármán

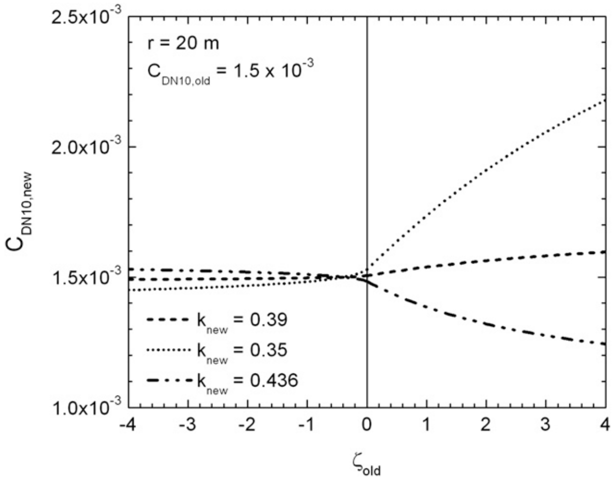

FIG. 9. As in Fig. 7, but $r = 20$ m.

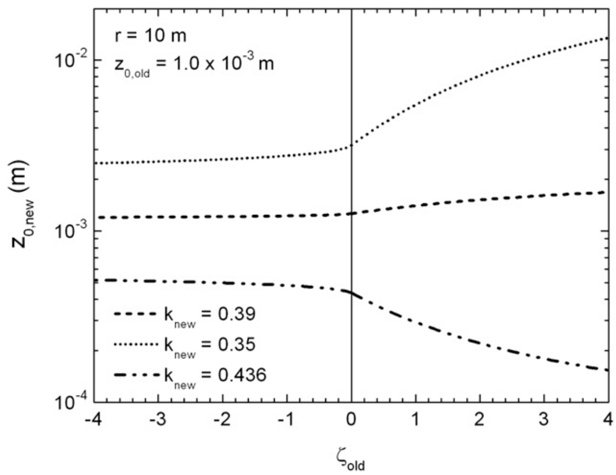

FIG. 10. New values of the roughness length, $z_{0,\mathrm{new}}$, computed from (6.11) for three new values of the von Kármán constant and for $k_{\mathrm{old}} = 0.40$. The original roughness length, $z_{0,\mathrm{old}}$, and measurement height, r, are assumed to be 1.0×10^{-3} m and 10 m, respectively; $z_{0,\mathrm{new}}$ also depends on the stratification during the original measurements, ζ_{old}.

constant other than 0.40—0.41 is another common choice (e.g., Dyer and Hicks 1970; Wieringa 1974; Garratt 1977; Resio et al. 2004)—they would be systematically different than $C_{\mathrm{DN10,old}}$ values based on $k_{\mathrm{old}} = 0.40$. The measurement height and the original stratification would determine how different, but (6.6) could be adapted to make the values comparable.

Because of (6.3), if C_{DN10} must be modified, z_0 must be also. Equation (6.3) yields expressions for old and new values of z_0:

$$z_{0,\mathrm{old}} = 10\ \exp(-k_{\mathrm{old}} C_{\mathrm{DN10,old}}^{-1/2}) \quad \text{and} \tag{6.7a}$$

$$z_{0,\mathrm{new}} = 10\ \exp(-k_{\mathrm{new}} C_{\mathrm{DN10,new}}^{-1/2}), \tag{6.7b}$$

where $z_{0,\mathrm{old}}$ and $z_{0,\mathrm{new}}$ are in meters.

Because we ultimately want to convert $z_{0,\mathrm{old}}$ to $z_{0,\mathrm{new}}$, I write their ratio from (6.7):

$$\frac{z_{0,\mathrm{new}}}{z_{0,\mathrm{old}}} = \exp(-k_{\mathrm{new}} C_{\mathrm{DN10,new}}^{-1/2} + k_{\mathrm{old}} C_{\mathrm{DN10,old}}^{-1/2}). \tag{6.8}$$

Using (6.5) to substitute here for $C_{\mathrm{DN10,new}}$ gives $z_{0,\mathrm{new}}$ in terms of old values:

$$\frac{z_{0,\mathrm{new}}}{z_{0,\mathrm{old}}} = \exp\left\{-(k_{\mathrm{new}} - k_{\mathrm{old}}) C_{\mathrm{DN10,old}}^{-1/2} - \left(\frac{k_{\mathrm{new}} - k_{\mathrm{old}}}{k_{\mathrm{old}}\, k_{\mathrm{new}}}\right)[\ln(r/10) - \psi_{m,\mathrm{old}}(\zeta_{\mathrm{old}})]\right\}. \tag{6.9}$$

Substituting (6.3) here yields

$$\frac{z_{0,\mathrm{new}}}{z_{0,\mathrm{old}}} = \exp\left\{\left(1 - \frac{k_{\mathrm{new}}}{k_{\mathrm{old}}}\right)[\ln(10/z_{0,\mathrm{old}}) + \ln(r/10) - \psi_{m,\mathrm{old}}(\zeta_{\mathrm{old}})]\right\}. \tag{6.10}$$

Hence, finally,

$$z_{0,\mathrm{new}} = z_{0,\mathrm{old}}^{k_{\mathrm{new}}/k_{\mathrm{old}}}\, r^{1-(k_{\mathrm{new}}/k_{\mathrm{old}})} \times \exp\left[\left(\frac{k_{\mathrm{new}}}{k_{\mathrm{old}}} - 1\right)\psi_{m,\mathrm{old}}(\zeta_{\mathrm{old}})\right]. \tag{6.11}$$

As we saw with $C_{\mathrm{DN10,new}}$, $z_{0,\mathrm{new}}$ depends on the originally measured roughness length, $z_{0,\mathrm{old}}$; on the height of the original measurements, r; and on the original stratification, ζ_{old}. If the original stratification was neutral, however, $\psi_{m,\mathrm{old}}(\zeta_{\mathrm{old}}) = 0$ in (6.11); and $z_{0,\mathrm{new}}$ does not need to be corrected for stratification. In contrast to modifying $C_{\mathrm{DN10,new}}$, the original measurement height never drops out of (6.11) and always affects $z_{0,\mathrm{new}}$. Figures 10 and 11 show some sample calculations of $z_{0,\mathrm{new}}$ that illustrate these dependencies.

Figure 10 depicts modifications in $z_{0,\mathrm{new}}$ for a standard reference height of 10 m and an assumed value of $z_{0,\mathrm{old}} = 1.0 \times 10^{-3}$ m. For a small change in the von Kármán constant—say, from $k_{\mathrm{old}} = 0.40$ to $k_{\mathrm{new}} = 0.39$—$z_{0,\mathrm{new}}$ is

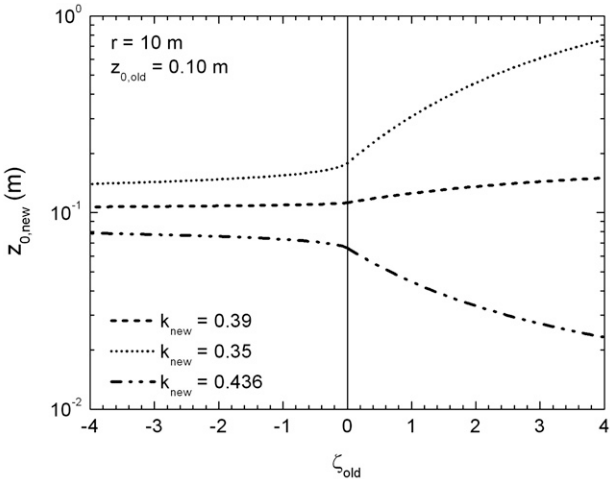

FIG. 11. As in Fig. 10, but $z_{0,\mathrm{old}} = 0.10$ m.

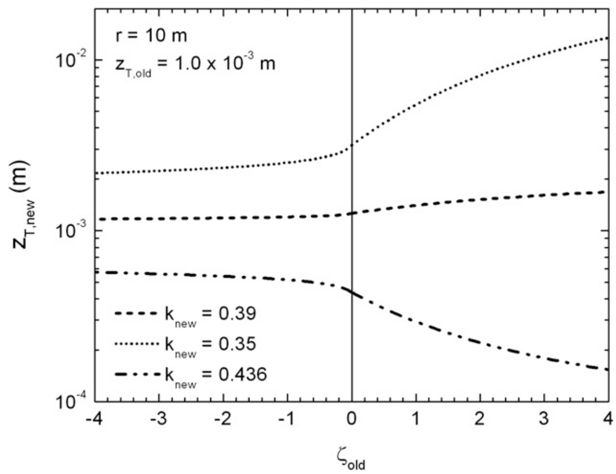

FIG. 12. New values of the roughness length for temperature, $z_{T,\mathrm{new}}$, computed from (7.2) for three new values of the von Kármán constant and for $k_{\mathrm{old}} = 0.40$. The original roughness length, $z_{T,\mathrm{old}}$, and measurement height, r, are assumed to be 1.0×10^{-3} m and 10 m, respectively; $z_{T,\mathrm{new}}$ also depends on the stratification during the original measurements, ζ_{old}.

only slightly larger than $z_{0,\mathrm{old}}$. If k_{new} undergoes a larger revision—to 0.35 or to 0.436—$z_{0,\mathrm{new}}$ changes by a factor of about 2.5 in unstable stratification and by as much as a factor of 10 in stable stratification. If k_{new} is smaller than k_{old}, $z_{0,\mathrm{new}}$ will be larger than $z_{0,\mathrm{old}}$. If k_{new} is larger than k_{old}, $z_{0,\mathrm{new}}$ will be smaller than $z_{0,\mathrm{old}}$. Notice, moreover, that for no combination of k_{new} and ζ_{old} in Fig. 10 does $z_{0,\mathrm{new}}$ equal $z_{0,\mathrm{old}}$. This fact is a consequence of the exponentiation of both $z_{0,\mathrm{old}}$ and r in (6.11).

For $z_{0,\mathrm{old}}$ as depicted in Fig. 10 but with $r = 5$ m (not shown), all $z_{0,\mathrm{new}}$ values move closer to $z_{0,\mathrm{old}}$. This improvement, however, is very slight because the exponent on r in (6.11) is small—0.09–0.125 for the assumed range of k_{new}.

Figure 11 is like Fig. 10, but here $z_{0,\mathrm{old}} = 0.10$ m, 100 times larger than in Fig. 10. The relative change between $z_{0,\mathrm{new}}$ and $z_{0,\mathrm{old}}$ is smaller in this figure than in Fig. 10. That effect results because relative change goes as $z_{0,\mathrm{old}}^{(k_{\mathrm{new}}/k_{\mathrm{old}})-1}$. Hence, as $z_{0,\mathrm{old}}$ increases, the relative change decreases. Nevertheless, still for this set of calculations, $z_{0,\mathrm{new}}$ can be quite different from $z_{0,\mathrm{old}}$ depending on how much the von Kármán constant is changed and what the original stratification was.

7. Scalar transfer coefficients and scalar roughness lengths

Equations (6.1) imply that the transfer coefficients C_{Dr}, C_{Hr}, and C_{Er} derive strictly from data and are independent of the assumed value of the von Kármán constant. Hence, as with $z_{0,\mathrm{new}}$, we can use (6.2) to evaluate new values of the scalar roughness lengths. With the roughness length for temperature, z_T, as an example, I obtain from (6.2b) a relation between old and new values:

$$\frac{C_{Hr}}{C_{\mathrm{Dr}}^{1/2}} = \frac{k_{\mathrm{old}}}{\ln(r/z_{T,\mathrm{old}}) - \psi_{h,\mathrm{old}}(\zeta_{\mathrm{old}})} = \frac{k_{\mathrm{new}}}{\ln(r/z_{T,\mathrm{new}}) - \psi_{h,\mathrm{new}}(\zeta_{\mathrm{new}})}. \tag{7.1}$$

With the identity (A.8b) inserted, (7.1) easily coverts to an expression for $z_{T,\mathrm{new}}$:

$$z_{T,\mathrm{new}} = z_{T,\mathrm{old}}^{k_{\mathrm{new}}/k_{\mathrm{old}}} r^{1-(k_{\mathrm{new}}/k_{\mathrm{old}})} \times \exp\left[\left(\frac{k_{\mathrm{new}}}{k_{\mathrm{old}}} - 1\right)\psi_{h,\mathrm{old}}(\zeta_{\mathrm{old}})\right], \tag{7.2}$$

which has the same form as (6.11). Consequently, most of the statements that I made about the relationship between $z_{0,\mathrm{new}}$ and $z_{0,\mathrm{old}}$ are accurate also for $z_{T,\mathrm{new}}$ and $z_{T,\mathrm{old}}$.

Figure 12 shows one example of $z_{T,\mathrm{new}}$ to illustrate how it is related to $z_{T,\mathrm{old}}$. This figure repeats similar conditions from Fig. 10: $r = 10$ m and $z_{T,\mathrm{old}} = 1.0 \times 10^{-3}$ m. On the unstable side of Fig. 12, $z_{T,\mathrm{new}}$ is always closer to $z_{T,\mathrm{old}}$ than $z_{0,\mathrm{new}}$ is to $z_{0,\mathrm{old}}$ in Fig. 10 because $\psi_{h,\mathrm{old}}(\zeta_{\mathrm{old}})$ is always larger than $\psi_{m,\mathrm{old}}(\zeta_{\mathrm{old}})$ in unstable stratification. On the stable side, the $z_{T,\mathrm{new}}$ and $z_{0,\mathrm{new}}$ values are the same in Figs. 10 and 12 because $\psi_{m,\mathrm{old}}(\zeta_{\mathrm{old}}) = \psi_{h,\mathrm{old}}(\zeta_{\mathrm{old}})$ here [see (A.5)]. Finally, as with $z_{0,\mathrm{old}}$ and $z_{0,\mathrm{new}}$, for no combination of k_{new} and ζ_{old} does $z_{T,\mathrm{new}}$ equal $z_{T,\mathrm{old}}$.

Because most authorities use the same stratification correction, ψ_h, for both temperature and humidity (for both sensible and latent heat transfer), (7.2) is also the equation for relating old and new values of the roughness lengths for humidity, $z_{Q,\mathrm{old}}$ and $z_{Q,\mathrm{new}}$.

Next, let us turn to developing a relation between old and new values of the scalar transfer coefficients. As with the drag coefficient, (6.3), most prefer to work with the neutral-stability transfer coefficient at a standard reference height of 10 m. From (6.2), with sensible heat again as the example, this quantity is

$$C_{HN10} = \frac{kC_{DN10}^{1/2}}{\ln(10/z_T)}, \quad (7.3)$$

where z_T must be in meters. With some manipulations, I convert (7.3) into an expression for the new value of the scalar transfer coefficient:

$$C_{HN10,new} = \frac{C_{DN10,new}^{1/2}}{C_{DN10,new}^{-1/2} - k_{new}^{-1}\ln(z_{T,new}/z_{0,new})}. \quad (7.4)$$

The ratio $z_{T,new}/z_{0,new}$ is interesting in itself (e.g., Chamberlain 1966; Garratt and Hicks 1973; Brutsaert 1982, 121–124) and merits a digression. Combining (6.11) and (7.2) produces

$$\frac{z_{T,new}}{z_{0,new}} = \left(\frac{z_{T,old}}{z_{0,old}}\right)^{k_{new}/k_{old}} \exp\left\{\left(\frac{k_{new}}{k_{old}} - 1\right) \times [\psi_{h,old}(\zeta_{old}) - \psi_{m,old}(\zeta_{old})]\right\}. \quad (7.5)$$

Figure 13 shows sample calculations of this ratio.

While Figs. 10–12 show some dramatic changes in $z_{0,new}$ and $z_{T,new}$ as a consequence of changing the von Kármán constant, Fig. 13 demonstrates that the ratio $z_{T,new}/z_{0,new}$ is resistant to change. One reason is that $z_{0,new}$ and $z_{T,new}$ have similar functional forms; hence, taking their ratio cancels the dependence on measurement height r, for example. Also, because the $\psi_{m,old}$ and $\psi_{h,old}$ functions that I use in stable stratification are the same, Fig. 13 and (7.5) show that $z_{T,new}/z_{0,new}$ does not depend on stratification on the stable side of the plot.

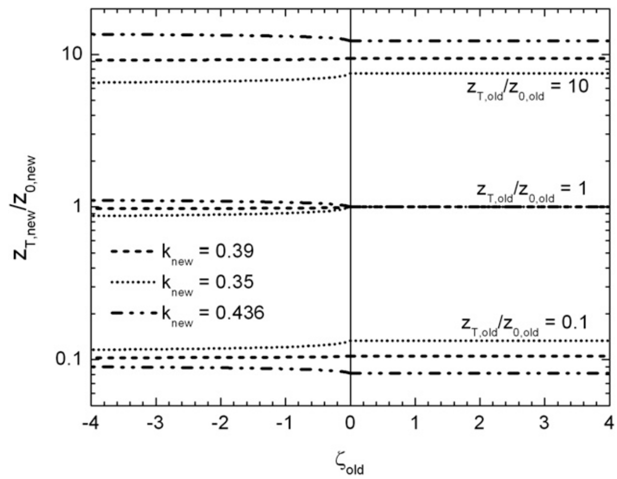

FIG. 13. The roughness ratio $z_{T,new}/z_{0,new}$ for three different values of k_{new} and for three different values for the original ratio, $z_{T,old}/z_{0,old}$, calculated from (7.5). The original von Kármán constant, k_{old}, is assumed to be 0.40. The ratio also depends on the original stratification, ζ_{old}.

Now back to evaluating $C_{HN10,new}$. Substituting $C_{DN10,new}^{1/2}$ from (6.5) and $z_{T,new}/z_{0,new}$ from (7.5) into (7.4) gives

$$C_{HN10,new} = \left\{\frac{1}{C_{DN10,old}^{-1/2} + \left(\frac{k_{new} - k_{old}}{k_{new}k_{old}}\right)[\ln(r/10) - \psi_{m,old}(\zeta_{old})]}\right\} \times \left\{\frac{1}{C_{DN10,old}^{-1/2} - k_{old}^{-1}\ln(z_{T,old}/z_{0,old}) + \left(\frac{k_{new} - k_{old}}{k_{new}k_{old}}\right)[\ln(r/10) - \psi_{h,old}(\zeta_{old})]}\right\}. \quad (7.6)$$

Using (7.3), I could also have formulated this in terms of $z_{0,old}$ rather than $C_{DN10,old}$, but the current form is less cumbersome.

From (7.6), we see that $C_{HN10,new}$ is the most complex of the new functions that I have derived. It depends on $C_{DN10,old}$, on the original measurement height, on the old and new values of the von Kármán constant, on the ratio $z_{T,old}/z_{0,old}$, and on the original stratification through both $\psi_{m,old}$ and $\psi_{h,old}$. This same expression will predict the neutral-stability, 10-m value of the latent heat transfer coefficient, $C_{EN10,new}$: just replace $z_{T,old}$ with $z_{Q,old}$.

Analogous to (7.4), $C_{HN10,old}$ takes the form

$$C_{HN10,old} = \frac{C_{DN10,old}^{1/2}}{C_{DN10,old}^{-1/2} - k_{old}^{-1}\ln(z_{T,old}/z_{0,old})}. \quad (7.7)$$

For neutral stratification [where $\psi_{m,old}(\zeta_{old}) = \psi_{h,old}(\zeta_{old}) = 0$] and an original measurement height

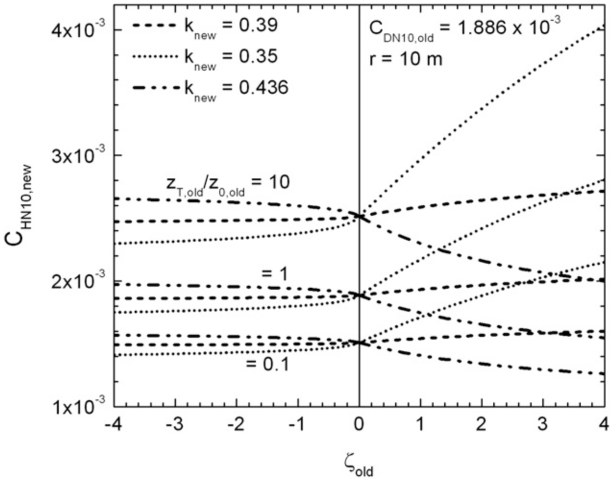

FIG. 14. Corrected values of the sensible heat transfer coefficient, $C_{\mathrm{HN10,new}}$, computed from (7.6). Original conditions are assumed to be $k_{\mathrm{old}} = 0.40$, $r = 10$ m, $C_{\mathrm{DN10,old}} = 1.886 \times 10^{-3}$ ($z_{0,\mathrm{old}} = 1.0 \times 10^{-3}$ m), and stratification (ζ_{old}) as plotted. For the range of $z_{T,\mathrm{old}}/z_{0,\mathrm{old}}$ values shown—10, 1, and 0.1—$C_{\mathrm{HN10,old}}$ is, respectively, 2.515×10^{-3}, 1.886×10^{-3}, and 1.509×10^{-3}.

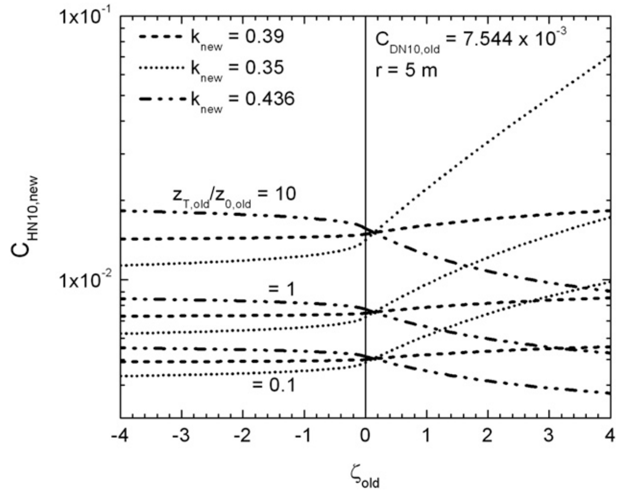

FIG. 15. As in Fig. 14, but $r = 5$ m and $C_{\mathrm{DN10,old}} = 7.544 \times 10^{-3}$ ($z_{0,\mathrm{old}} = 0.10$ m). Consequently, for $z_{T,\mathrm{old}}/z_{0,\mathrm{old}}$ values of 10, 1, and 0.1, $C_{\mathrm{HN10,old}}$ is, respectively, 1.509×10^{-2}, 7.544×10^{-3}, and 5.030×10^{-3}. Notice that the vertical axis is logarithmic in this plot.

r of 10 m, the right-hand side of (7.6) simplifies to the right-hand side of (7.7). That is, for these original conditions, $C_{\mathrm{HN10,new}} = C_{\mathrm{HN10,old}}$. For other values of r and the stratification, however, $C_{\mathrm{HN10,new}}$ can differ greatly from $C_{\mathrm{HN10,old}}$. Figures 14 and 15 show examples of these results.

In Fig. 14, for which $r = 10$ m, $C_{\mathrm{DN10,old}} = 1.886 \times 10^{-3}$ corresponds to $z_{0,\mathrm{old}} = 1.0 \times 10^{-3}$ m, as in Fig. 10. The plot depicts the same values for k_{old} and k_{new} that I have been considering but shows how $C_{\mathrm{HN10,new}}$ also depends on the three values of $z_{T,\mathrm{old}}/z_{0,\mathrm{old}}$: 10, 1, and 0.1. For these three ratios, $C_{\mathrm{HN10,old}}$ is, respectively, 2.515×10^{-3}, 1.886×10^{-3}, and 1.509×10^{-3}. The three curves in each $z_{T,\mathrm{old}}/z_{0,\mathrm{old}}$ set intersect the $\zeta_{\mathrm{old}} = 0$ axis at these values. Consequently, Fig. 14 shows that $C_{\mathrm{HN10,new}}$ does not deviate much from $C_{\mathrm{NH10,old}}$ on the unstable side of the plot. For the k_{new} values shown, however, $C_{\mathrm{HN10,new}}$ can change by as much as 60% if the original data were collected in stable stratification.

Figure 15 shows how $C_{\mathrm{HN10,new}}$ behaves for $r = 5$ m and for $C_{\mathrm{DN10,old}}$ increased to 7.544×10^{-3} ($z_{0,\mathrm{old}} = 0.10$ m). The three curves in each $z_{T,\mathrm{old}}/z_{0,\mathrm{old}}$ set no longer intersect on the $\zeta_{\mathrm{old}} = 0$ axis. That intersection represents $C_{\mathrm{HN10,old}}$ and is offset toward the stable side because it occurs where $\ln(r/10) = \psi_{m,\mathrm{old}}(\zeta_{\mathrm{old}})$ [because $\psi_{m,\mathrm{old}}(\zeta_{\mathrm{old}}) = \psi_{h,\mathrm{old}}(\zeta_{\mathrm{old}})$ in stable stratification]. Because of the larger drag coefficient, the $C_{\mathrm{HN10,new}}$ values spread according to the von Kármán constant much more widely than in Fig. 14. If the von Kármán constant is changed to $k_{\mathrm{new}} = 0.39$, the required modification in $C_{\mathrm{HN10,new}}$ is still less than typical random scatter in the data for all original stratifications. But if the von Kármán constant is changed by about 10%—to 0.35 or 0.436—the required change in $C_{\mathrm{HN10,new}}$ can be large: as much as 20% in unstable stratification and well over 100% if the original measurements were in very stable stratification.

8. Dissipation method

The dissipation method uses the budget equations for turbulent kinetic energy and scalar variance and either direct or inertial-subrange measurements of the dissipation rates of turbulent kinetic energy (ε) and temperature (N_θ) and humidity (N_q) variances to estimate the fluxes of momentum and sensible and latent heat (e.g., Taylor 1961; Large and Pond 1981, 1982; Fairall and Larsen 1986; Hsieh et al. 1996; Yelland and Taylor 1996). Because the von Kármán constant appears in these budget equations, fluxes derived from the dissipation method may need to be revised if the von Kármán constant is changed.

Flux calculations based on the dissipation method usually assume the simple production-equals-dissipation form of the budget equations (e.g., Andreas 1987a):

$$u_*^2 \frac{\partial U}{\partial z} + \frac{g}{\Theta_v} \overline{w\theta_v} = \varepsilon, \quad (8.1a)$$

$$2u_*\theta_* \frac{\partial \Theta}{\partial z} = N_\theta, \quad (8.1b)$$

$$2u_*q_* \frac{\partial Q}{\partial z} = N_q. \quad (8.1c)$$

In (8.1c), $\partial Q/\partial z$ is the vertical gradient in mean specific humidity.

The ε, N_θ, and N_q in (8.1), the dissipation rates, are the fundamental data here. Typically, these come from measurements in the inertial subrange of the velocity spectrum and in the inertial-convective subrange of the temperature and humidity spectra. Such dissipation rates result in the so-called inertial-dissipation method. The dissipation rates can, however, be measured directly (e.g., Paquin and Pond 1971; Champagne et al. 1977; Williams and Paulson 1977; Oncley et al. 1996); such measurements result in the direct dissipation method.

The von Kármán constant enters the analysis because we must rewrite (8.1) in terms of the Monin–Obukhov similarity functions to continue the calculations:

$$\varepsilon = \frac{u_*^3}{kz}[\phi_m(\zeta) - \zeta], \tag{8.2a}$$

$$N_\theta = \frac{2u_*\theta_*^2}{kz}\phi_h(\zeta), \quad \text{and} \tag{8.2b}$$

$$N_q = \frac{2u_*q_*^2}{kz}\phi_h(\zeta). \tag{8.2c}$$

Remember, the left-hand sides here represent the measurements; we seek the flux scales u_*, θ_*, and q_*. The three equations are coupled and must be solved iteratively because ζ contains the fluxes.

As in some of the earlier analyses in this paper, I can relate old and new fluxes by equating the measurements that remain unchanged under a conversion in the von Kármán constant. For the turbulent kinetic energy budget, (8.2a), this assumption yields

$$\begin{aligned} z\,\varepsilon &= \frac{u_{*,\mathrm{old}}^3}{k_{\mathrm{old}}}[\phi_{m,\mathrm{old}}(\zeta_{\mathrm{old}}) - \zeta_{\mathrm{old}}] \\ &= \frac{u_{*,\mathrm{new}}^3}{k_{\mathrm{new}}}[\phi_{m,\mathrm{new}}(\zeta_{\mathrm{new}}) - \zeta_{\mathrm{new}}]. \end{aligned} \tag{8.3}$$

Hence,

$$\frac{u_{*,\mathrm{new}}}{u_{*,\mathrm{old}}} = \left(\frac{k_{\mathrm{new}}}{k_{\mathrm{old}}}\right)^{1/3}\left[\frac{\phi_{m,\mathrm{old}}(\zeta_{\mathrm{old}}) - \zeta_{\mathrm{old}}}{\phi_{m,\mathrm{old}}(\zeta_{\mathrm{old}}) - \left(\dfrac{k_{\mathrm{new}}}{k_{\mathrm{old}}}\right)\zeta_{\mathrm{old}}}\right]^{1/3}, \tag{8.4}$$

which incorporates (2.3) and (2.24a) to represent the right-hand side in terms of old values.

Figure 16 shows this u_* ratio as a function of the original stratification for the usual three values of k_{new}.

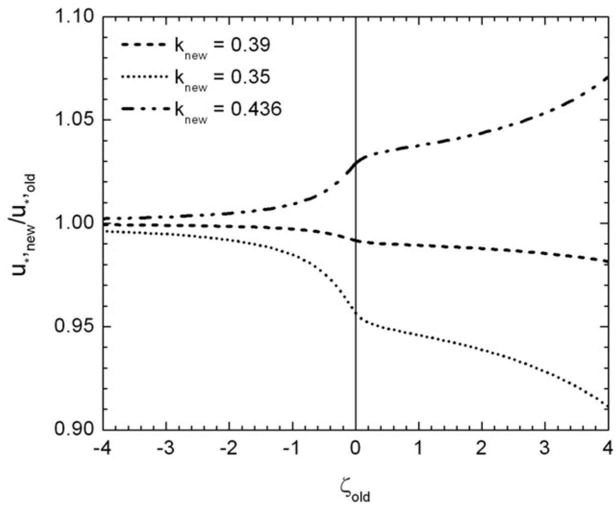

FIG. 16. The relationship between old and new values of the friction velocity (u_*), where the dissipation method yielded $u_{*,\mathrm{old}}$. The results come from (8.4) with $k_{\mathrm{old}} = 0.40$ for each k_{new} curve shown; ζ_{old} is the stratification during the original measurements.

For most values of ζ_{old} in unstable stratification, $u_{*,\mathrm{new}}$ and $u_{*,\mathrm{old}}$ are within 1% of each other for all reasonable changes in the von Kármán constant. In contrast to most of the other results, however, $u_{*,\mathrm{new}}$ and $u_{*,\mathrm{old}}$ differ increasingly as unstable stratification goes toward neutral stratification: at $\zeta_{\mathrm{old}} = 0$, $u_{*,\mathrm{new}}$ and $u_{*,\mathrm{old}}$ differ by as much as 4%. In stable stratification, the difference increases even more—to more than 7% at the extreme right edge of Fig. 16, the very stable side of the plot.

While percentages in the ranges mentioned above are smaller than typical uncertainties in measurements or estimates of u_*, the above results are bias errors rather than random errors. As such, they are more serious if left uncorrected but are more easily corrected than random experimental errors.

A method for modifying the heat fluxes obtained by the dissipation method likewise is derived from the scalar variance budgets. With sensible heat as the example [i.e., (8.2b)], I again equate the unchanged measurements:

$$\begin{aligned} zN_\theta &= \frac{2u_{*,\mathrm{old}}\theta_{*,\mathrm{old}}^2}{k_{\mathrm{old}}}\phi_{h,\mathrm{old}}(\zeta_{\mathrm{old}}) \\ &= \frac{2u_{*,\mathrm{new}}\theta_{*,\mathrm{new}}^2}{k_{\mathrm{new}}}\phi_{h,\mathrm{new}}(\zeta_{\mathrm{new}}). \end{aligned} \tag{8.5}$$

Because $\phi_{h,\mathrm{old}}(\zeta_{\mathrm{old}}) = \phi_{h,\mathrm{new}}(\zeta_{\mathrm{new}})$, (8.5) reduces to

$$\frac{\theta_{*,\mathrm{new}}}{\theta_{*,\mathrm{old}}} = \left(\frac{k_{\mathrm{new}}}{k_{\mathrm{old}}}\right)^{1/2}\left(\frac{u_{*,\mathrm{old}}}{u_{*,\mathrm{new}}}\right)^{1/2}. \tag{8.6}$$

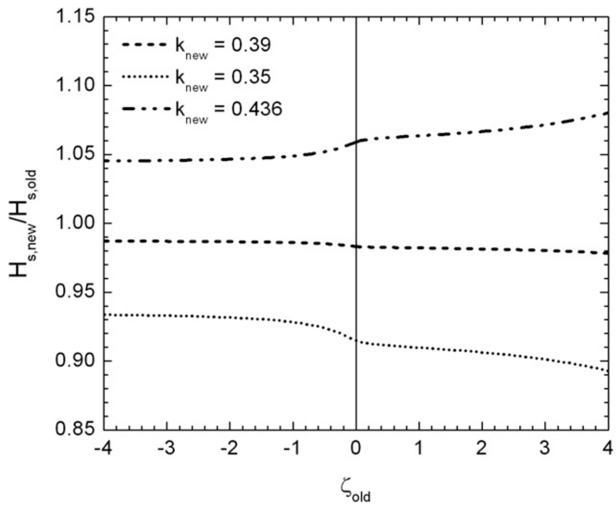

FIG. 17. The relationship between old and new values of the sensible heat flux (H_s), where $H_{s,\text{old}}$ is assumed to have derived from the dissipation method. The results come from (8.8) with $k_{\text{old}} = 0.40$ for each value of k_{new} shown; ζ_{old} is the stratification during the original measurements.

In turn, (8.6) converts to an equation for the sensible heat flux [cf. (6.1b)]:

$$\frac{H_{s,\text{new}}}{H_{s,\text{old}}} = \frac{-\rho c_p u_{*,\text{new}}\,\theta_{*,\text{new}}}{-\rho c_p u_{*,\text{old}}\,\theta_{*,\text{old}}} = \left(\frac{k_{\text{new}}}{k_{\text{old}}}\right)^{1/2}\left(\frac{u_{*,\text{new}}}{u_{*,\text{old}}}\right)^{1/2}. \tag{8.7}$$

Finally, substituting (8.4) into (8.7) gives

$$\frac{H_{s,\text{new}}}{H_{s,\text{old}}} = \left(\frac{k_{\text{new}}}{k_{\text{old}}}\right)^{2/3}\left[\frac{\phi_{m,\text{old}}(\zeta_{\text{old}}) - \zeta_{\text{old}}}{\phi_{m,\text{old}}(\zeta_{\text{old}}) - \left(\frac{k_{\text{new}}}{k_{\text{old}}}\right)\zeta_{\text{old}}}\right]^{1/6}. \tag{8.8}$$

Figure 17 shows this ratio. The ratio $H_{s,\text{new}}/H_{s,\text{old}}$ has a much weaker dependence on original stratification than does $u_{*,\text{new}}/u_{*,\text{old}}$ because of the 1/6 power on the ζ_{old} term [as compared with the 1/3 power in (8.4)]. But $H_{s,\text{new}}/H_{s,\text{old}}$ has a stronger dependence on $k_{\text{new}}/k_{\text{old}}$ because of the 2/3 power on that term in (8.8) as compared with the 1/3 power in (8.4). In overall effect, the $H_{s,\text{new}}/H_{s,\text{old}}$ ratio varies by only a few percent over the stratification range depicted in Fig. 17. Thus, a constant multiplicative correction of $(k_{\text{new}}/k_{\text{old}})^{2/3}$ is adequate for converting $H_{s,\text{old}}$ to $H_{s,\text{new}}$ for all original values of the stratification parameter ζ_{old}.

Because (8.2c) implies that the same similarity function ϕ_h is appropriate in both the temperature and humidity variance budgets, (8.8) and Fig. 17 also represent the ratio of latent heat fluxes, $H_{L,\text{new}}/H_{L,\text{old}}$.

Often, inertial-dissipation estimates of the momentum and heat fluxes are used in equations like (6.1) and (6.2) to evaluate transfer coefficients and roughness lengths (e.g., Large and Pond 1981, 1982; Anderson 1987, 1993; Guest and Davidson 1991; Yelland et al. 1998). The resulting transfer coefficients and roughness lengths thus suffer doubly under a change in the von Kármán constant because that new value must be incorporated into both steps of the analysis. The equations that I have derived will let the interested reader solve this puzzle.

9. Discussion

For most of these analyses, I ultimately had to assume a particular form for one or more of the Monin–Obukhov similarity functions. In a sense, this specificity limits the results because my choices might not be your preferred similarity functions. Fortunately, in unstable stratification at least, published similarity functions agree in general behavior if not in every detail.

In stable stratification in contrast, reported functions vary widely in almost all details. Hence, using functions other than my choices may change some of the results presented here. One particular set of choices that deserves further discussion is the Holtslag and De Bruin (1988) functions for ϕ_m, ϕ_h, ψ_m, and ψ_m. Because $\phi_m = \phi_h$ and $\psi_m = \psi_h$ for these functions, the plot of $z_{T,\text{new}}/z_{0,\text{new}}$ in Fig. 13 shows no stratification dependence in stable stratification. Although others have reported ϕ_m and ϕ_h and ψ_m and ψ_h functions that are equal in stable stratification, especially if ϕ_m and ϕ_h are formulated as log-linear relations (e.g., Webb 1970; Dyer 1974; Large and Pond 1982), other modern functions tend to suggest different ϕ_m and ϕ_h functions. For example, Beljaars and Holtslag (1991) updated the Holtslag and De Bruin functions such that their new functions have the same complex form but ϕ_m and ϕ_h are not equal. And just recently, Grachev et al. (2007a) presented new and unequal ϕ_m and ϕ_h functions that have proper asymptotic behavior in very stable stratification.

I simply could not survey all the published similarity functions. The equations that I have derived, though, allow interested readers to evaluate how changing the von Kármán constant will alter results based on the similarity functions of their choice.

10. Conclusions

Credible evidence exists from measurements in both the laboratory (e.g., Österlund et al. 2000; Perry et al. 2001; Nagib et al. 2004) and the atmospheric surface layer (e.g., Dyer and Bradley 1982; Kondo and Sato 1982; Frenzen and Vogel 1995a,b; Andreas et al. 2006) that the von Kármán constant is nearer to 0.39 than to the canonical value of 0.40. I have shown how to modify existing similarity functions to implement this new value. In all the examples, the original form of the similarity function remains after the modification; only some empirical coefficients change.

For variables derived from similarity theory and used in modeling the atmospheric surface layer, however, modifications are not as straightforward. I refer here to the transfer coefficients C_{DN10}, C_{HN10}, and C_{EN10} and the roughness lengths z_0, z_T, and z_Q. Literature values of these that were obtained assuming $k_{old} = 0.40$ may need to be modified to be compatible for use with a new value of the von Kármán constant. The magnitude of the modification generally depends on the new value of the von Kármán constant and on the original measurement height, stratification, and value of the variable in question. Variables originally obtained in stable stratification generally require larger relative changes to be compatible with k_{new} than variables originally obtained in unstable stratification. Because of this dependence on stratification, variables may need to be modified one at a time rather then en masse because the stratification generally varies during the course of an experiment.

Fortunately, the approach to modifying transfer coefficients and roughness lengths that my analysis lays out may not often be necessary. For a von Kármán constant that changes only from $k_{old} = 0.40$ to $k_{new} = 0.39$, Figs. 7–15 suggest that modifying transfer coefficients and roughness lengths is probably unnecessary. For the stability range $-4 \leq \zeta \leq 4$, the changes in these variables required by setting $k_{new} = 0.39$ are smaller than the experimental scatter in original measurements of these variables.

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APPENDIX

Other Function Closely Related to ϕ_m and ϕ_h

a. Profile functions

Panofsky (1963; also Paulson 1970) demonstrates how to integrate (2.4) to obtain expressions for the surface layer profiles of average wind speed and potential temperature:

$$U_{old}(z) = \frac{u_*}{k_{old}}[\ln(z/z_{0,old}) - \psi_{m,old}(\zeta_{old})], \tag{A.1a}$$

$$\Theta_{old}(z) = \Theta_s + \frac{\theta_*}{k_{old}}[\ln(z/z_{T,old}) - \psi_{h,old}(\zeta_{old})]. \tag{A.1b}$$

This section focuses on converting the stratification corrections $\psi_{m,old}$ and $\psi_{h,old}$ to forms that reflect a new von Kármán constant, $\psi_{m,new}$ and $\psi_{h,new}$. A general ψ function derives from a general ϕ function through (Panofsky 1963)

$$\psi(\zeta) = \int_0^{\zeta} \frac{1 - \phi(\xi)}{\xi} d\xi, \tag{A.2}$$

where ξ is again a variable of integration.

Equations (A.1) also define other new terms. The $z_{0,old}$ and $z_{T,old}$ are, respectively, the roughness lengths for wind speed and temperature profiles that were obtained using the old value of the von Kármán constant. Also in (A.1), U_{old} and Θ_{old} imply that we are using (A.1) to estimate wind speed and temperature profiles based on measurements of u_* and θ_*. If, instead, the goal was to estimate u_* and θ_* from measurements of $U(z)$, $\Theta(z)$, and Θ_s, we would write (A.1) with the subscripts "old" removed from $U(z)$ and $\Theta(z)$ but added to u_* and θ_* to indicate which variables are data and which variables are calculated.

With (A.2) and the ϕ functions used in section 3, evaluating the ψ functions is straightforward. In unstable stratification, these are

$$\psi_{m,old}(\zeta_{old}) = 2\ln\left(\frac{1 + x_{old}}{2}\right) + \ln\left(\frac{1 + x_{old}^2}{2}\right) - 2\arctan(x_{old}) + \frac{\pi}{2}, \tag{A.3a}$$

$$\psi_{h,old}(\zeta_{old}) = 2\ln\left(\frac{1 + x_{old}^2}{2}\right), \tag{A.3b}$$

where

$$x_{old} = (1 - \gamma_{u,old}\zeta_{old})^{1/4}; \tag{A.3c}$$

and

$$\psi_{m,new}(\zeta_{new}) = 2\ln\left(\frac{1 + x_{new}}{2}\right) + \ln\left(\frac{1 + x_{new}^2}{2}\right) - 2\arctan(x_{new}) + \frac{\pi}{2}, \tag{A.4a}$$

$$\psi_{h,new}(\zeta_{new}) = 2\ln\left(\frac{1 + x_{new}^2}{2}\right), \tag{A.4b}$$

where

$$x_{new} = (1 - \gamma_{u,new}\zeta_{new})^{1/4}. \tag{A.4c}$$

For simplicity, (A.3c) and (A.4c) include the same multiplicative constant, γ_u, for both ϕ_m and ϕ_h, as Paulson (1970) found. Equations (2.11) and (2.17) show how $\gamma_{\mathrm{u,old}}$ and $\gamma_{\mathrm{u,new}}$ are related.

In stable stratification,

$$\psi_{m,\mathrm{old}}(\zeta_{\mathrm{old}}) = \psi_{h,\mathrm{old}}(\zeta_{\mathrm{old}}) = -\left[\alpha_{s,\mathrm{old}}\zeta_{\mathrm{old}} + \beta_{s,\mathrm{old}}\left(\zeta_{\mathrm{old}} - \frac{\gamma_{s,\mathrm{old}} - 1}{\delta_{s,\mathrm{old}}}\right)\exp(-\delta_{s,\mathrm{old}}\zeta_{\mathrm{old}}) + \frac{\beta_{s,\mathrm{old}}(\gamma_{s,\mathrm{old}} - 1)}{\delta_{s,\mathrm{old}}}\right], \quad (A.5)$$

and

$$\psi_{m,\mathrm{new}}(\zeta_{\mathrm{new}}) = \psi_{h,\mathrm{new}}(\zeta_{\mathrm{new}}) = -\left[\alpha_{s,\mathrm{new}}\zeta_{\mathrm{new}} + \beta_{s,\mathrm{new}}\left(\zeta_{\mathrm{new}} - \frac{\gamma_{s,\mathrm{old}} - 1}{\delta_{s,\mathrm{new}}}\right)\exp(-\delta_{s,\mathrm{new}}\zeta_{\mathrm{new}}) + \frac{\beta_{s,\mathrm{new}}(\gamma_{s,\mathrm{old}} - 1)}{\delta_{s,\mathrm{new}}}\right]. \quad (A.6)$$

Equation (2.22) relates the old and new values of α_s, β_s, and δ_s.

From (A.4c) and the definitions of $\gamma_{u,\mathrm{new}}$ and ζ_{new}, we see that

$$x_{\mathrm{new}} = \left[1 - \left(\frac{\gamma_{u,\mathrm{old}}k_{\mathrm{old}}}{k_{\mathrm{new}}}\right)(k_{\mathrm{new}}\widehat{\zeta})\right]^{1/4} = (1 - \gamma_{s,\mathrm{old}}k_{\mathrm{old}}\widehat{\zeta})^{1/4} = x_{\mathrm{old}}. \quad (A.7)$$

Consequently, for unstable stratification,

$$\psi_{m,\mathrm{new}}(\zeta_{\mathrm{new}}) = \psi_{m,\mathrm{old}}(\zeta_{\mathrm{old}}) \quad \text{and} \quad (A.8a)$$

$$\psi_{h,\mathrm{new}}(\zeta_{\mathrm{new}}) = \psi_{h,\mathrm{old}}(\zeta_{\mathrm{old}}). \quad (A.8b)$$

Similar algebra establishes that these same relations are true for stable stratification. These two equalities will prove useful in other analyses reported.

With $\psi_{m,\mathrm{new}}$ and $\psi_{h,\mathrm{new}}$ now identified, versions of (A.1) that reflect a new value of the von Kármán constant become

$$U_{\mathrm{new}}(z) = \frac{u_*}{k_{\mathrm{new}}}[\ln(z/z_{0,\mathrm{new}}) - \psi_{m,\mathrm{new}}(\zeta_{\mathrm{new}})], \quad (A.9a)$$

$$\Theta_{\mathrm{new}}(z) = \Theta_s + \frac{\theta_*}{k_{\mathrm{new}}}[\ln(z/z_{T,\mathrm{new}}) - \psi_{h,\mathrm{new}}(\zeta_{\mathrm{new}})]. \quad (A.9b)$$

In these, $z_{0,\mathrm{new}}$ and $z_{T,\mathrm{new}}$ are revised values of the roughness lengths that I relate to $z_{0,\mathrm{old}}$ and $z_{T,\mathrm{old}}$ in sections 6 and 7.

In summary, computing revised wind speed and potential temperature profiles from measurements of u_*, t_*, Θ_s, and $\widehat{\zeta}$ involves as many as five changes: introduce k_{new}; calculate ζ_{new} from $\widehat{\zeta}$; calculate the new stratification corrections, $\psi_{m,\mathrm{new}}$ and $\psi_{h,\mathrm{new}}$; and introduce proper new values of the roughness lengths for wind speed and temperature.

b. *Turbulent diffusivities*

The turbulent diffusivities for momentum and heat, K_m and K_h, respectively, are used in first-order closure to relate the vertical gradients in wind speed and potential temperature to the momentum and sensible heat fluxes, respectively. For the old value of the von Kármán constant, these functions are (e.g., Dyer 1974)

$$K_{m,\mathrm{old}}(\zeta_{\mathrm{old}}) = \frac{k_{\mathrm{old}}u_*z}{\phi_{m,\mathrm{old}}(\zeta_{\mathrm{old}})} \quad \text{and} \quad (A.10a)$$

$$K_{h,\mathrm{old}}(\zeta_{\mathrm{old}}) = \frac{k_{\mathrm{old}}u_*z}{\phi_{h,\mathrm{old}}(\zeta_{\mathrm{old}})}. \quad (A.10b)$$

Multiplying $\partial U/\partial z$, as defined by (2.4a), with $\rho K_{m,\mathrm{old}}$ yields the momentum flux ρu_*^2. Likewise, multiplying $\partial\Theta/\partial z$, as defined by (2.5a), by $-\rho c_p K_{h,\mathrm{old}}$ yields the sensible heat flux $-\rho c_p u_*\theta_*$.

Producing the same fluxes from the flux–gradient relations that reflect a new value of the von Kármán constant—namely from (2.4b) and 2.5b)—requires that

$$K_{m,\mathrm{new}}(\zeta_{\mathrm{new}}) = \frac{k_{\mathrm{new}}u_*z}{\phi_{m,\mathrm{new}}(\zeta_{\mathrm{new}})}, \quad (A.11a)$$

$$K_{h,\text{new}}(\zeta_{\text{new}}) = \frac{k_{\text{new}} u_* z}{\phi_{h,\text{new}}(\zeta_{\text{new}})}. \tag{A.11b}$$

Consequently, with K_m as the example, new and old values of the diffusivity are related by

$$K_{m,\text{new}}(\zeta_{\text{new}}) = \left(\frac{k_{\text{new}}}{k_{\text{old}}}\right)\left[\frac{\phi_{m,\text{old}}(\zeta_{\text{old}})}{\phi_{m,\text{new}}(\zeta_{\text{new}})}\right] K_{m,\text{old}}(\zeta_{\text{old}}). \tag{A.12}$$

Because of the equality (2.24a), this equation reduces to

$$K_{m,\text{new}}(\zeta_{\text{new}}) = \left(\frac{k_{\text{new}}}{k_{\text{old}}}\right) K_{m,\text{old}}(\zeta_{\text{old}}). \tag{A.13}$$

Similar algebra establishes that

$$K_{h,\text{new}}(\zeta_{\text{new}}) = \left(\frac{k_{\text{new}}}{k_{\text{old}}}\right) K_{h,\text{old}}(\zeta_{\text{old}}). \tag{A.14}$$

Consequently, with $k_{\text{old}} = 0.40$, $K_{m,\text{new}}$ and $K_{h,\text{new}}$ are, respectively, 0.975, 0.875, and 1.090 times $K_{m,\text{old}}$ and $K_{h,\text{old}}$ for k_{new} values of 0.39, 0.35, and 0.436. For the largest and smallest reasonable values for k_{new}, these are significant differences between old and new diffusivities. For $k_{\text{new}} = 0.39$, the difference is an almost negligible 2.5%.

REFERENCES

Anderson, R. J., 1987: Wind stress measurements over rough ice during the 1984 Marginal Ice Zone Experiment. *J. Geophys. Res.,* **92,** 6933–6941.

——, 1993: A study of wind stress and heat flux over the open ocean by the inertial-dissipation method. *J. Phys. Oceanogr.,* **23,** 2153–2161.

Andreas, E. L, 1987a: Spectral measurements in a disturbed boundary layer over snow. *J. Atmos. Sci.,* **44,** 1912–1939.

——, 1987b: On the Kolmogorov constants for the temperature–humidity cospectrum and the refractive index spectrum. *J. Atmos. Sci.,* **44,** 2399–2406.

——, 1988: Estimating C_n^2 over snow and sea ice from meteorological data. *J. Opt. Soc. Amer.,* **A5,** 481–495.

——, 1989: Two-wavelength method of measuring path-averaged turbulent surface heat fluxes. *J. Atmos. Oceanic Technol.,* **6,** 280–292.

——, 1998: The atmospheric boundary layer over polar marine surfaces. *Physics of Ice-Covered Seas,* M. Leppäranta, Ed., Vol. 2, Helsinki University Press, 715–773.

——, 2002: Parameterizing scalar transfer over snow and ice: A review. *J. Hydrometeor.,* **3,** 417–432.

——, R. J. Hill, J. R. Gosz, D. I. Moore, W. D. Otto, and A. D. Sarma, 1998: Statistics of surface-layer turbulence over terrain with metre-scale heterogeneity. *Bound.-Layer Meteor.,* **86,** 379–408.

——, K. J. Claffey, R. E. Jordan, C. W. Fairall, P. S. Guest, P. O. G. Persson, and A. A. Grachev, 2006: Evaluations of the von Kármán constant in the atmospheric surface layer. *J. Fluid Mech.,* **559,** 117–149.

Beljaars, A. C. M., and A. A. M. Holtslag, 1991: Flux parameterization over land surfaces for atmospheric models. *J. Appl. Meteor.,* **30,** 327–341.

Brutsaert, W. A., 1982: *Evaporation into the Atmosphere: Theory, History, and Applications*. D. Reidel, 299 pp.

Businger, J. A., and A. M. Yaglom, 1971: Introduction to Obukhov's paper on 'Turbulence in an atmosphere with a non-uniform temperature.' *Bound.-Layer Meteor.,* **2,** 3–6.

——, J. C. Wyngaard, Y. Izumi, and E. F. Bradley, 1971: Flux-profile relationships in the atmospheric surface layer. *J. Atmos. Sci.,* **28,** 181–189.

Chamberlain, A. C., 1966: Transport of gases to and from grass and grass-like surfaces. *Proc. Roy. Soc. London,* **A290,** 236–265.

Champagne, F. H., C. A. Friehe, J. C. LaRue, and J. C. Wyngaard, 1977: Flux measurements, flux estimation techniques, and fine-scale turbulence measurements in the unstable surface layer over land. *J. Atmos. Sci.,* **34,** 515–530.

Corrsin, S., 1951: On the spectrum of isotropic temperature fluctuations in an isotropic turbulence. *J. Appl. Phys.,* **22,** 469–473.

Davidson, K. L., T. M. Houlihan, C. W. Fairall, and G. E. Schacher, 1978: Observation of the temperature structure function parameter, C_T^2, over the ocean. *Bound.-Layer Meteor.,* **15,** 507–523.

De Bruin, H. A. R., W. M. L. Meijninger, A.-S. Smedman, and M. Magnusson, 2002: Displaced-beam small aperture scintillometer test. Part I: The WINTEX data-set. *Bound.-Layer Meteor.,* **105,** 129–148.

Dyer, A. J., 1974: A review of flux-profile relationships. *Bound.-Layer Meteor.,* **7,** 363–372.

——, and B. B. Hicks, 1970: Flux–gradient relationships in the constant flux layer. *Quart. J. Roy. Meteor. Soc.,* **96,** 715–721.

——, and E. F. Bradley, 1982: An alternative analysis of flux–gradient relationships at the 1976 ITCE. *Bound.-Layer Meteor.,* **22,** 3–19.

Edson, J. B., and C. W. Fairall, 1998: Similarity relationships in the marine atmospheric surface layer for terms in the TKE and scalar variance budgets. *J. Atmos. Sci.,* **55,** 2311–2328.

Fairall, C. W., and S. E. Larsen, 1986: Inertial-dissipation method and turbulent fluxes at the air-ocean interface. *Bound.-Layer Meteor.,* **34,** 287–301.

——, G. E. Schacher, and K. L. Davidson, 1980: Measurements of the humidity structure function parameters, C_q^2 and C_{Tq}, over the ocean. *Bound.-Layer Meteor.,* **19,** 81–92.

Frenzen, P., and C. A. Vogel, 1995a: On the magnitude and apparent range of variation of the von Karman constant in the atmospheric surface layer. *Bound.-Layer Meteor.,* **72,** 371–392.

——, and ——, 1995b: A further note "On the magnitude and apparent range of variation of the von Karman constant." *Bound.-Layer Meteor.,* **73,** 315–317.

Garratt, J. R., 1977: Review of drag coefficients over oceans and continents. *Mon. Wea. Rev.,* **105,** 915–929.

——, 1992: *The Atmospheric Boundary Layer*. Cambridge University Press, 316 pp.

——, and B. B. Hicks, 1973: Momentum, heat and water vapour transfer to and from natural and artificial surfaces. *Quart. J. Roy. Meteor. Soc.,* **99,** 680–687.

Grachev, A. A., E. L Andreas, C. W. Fairall, P. S. Guest, and P. O. G. Persson, 2007a: SHEBA flux-profile relationships in the stable atmospheric boundary layer. *Bound.-Layer Meteor.,* **124,** 315–333.

——, ——, ——, ——, and ——, 2007b: On the turbulent Prandtl number in the stable atmospheric surface layer. *Bound.-Layer Meteor.,* **125,** 329–341.

Green, A. E., K. J. McAneney, and M. S. Astill, 1994: Surface-layer scintillation measurements of daytime sensible heat and momentum fluxes. *Bound.-Layer Meteor.,* **68,** 357–373.

Guest, P. S., and K. L. Davidson, 1991: The aerodynamic roughness of different types of sea ice. *J. Geophys. Res.,* **96,** 4709–4721.

Haugen, D. A., J. C. Kaimal, and E. F. Bradley, 1971: An experimental study of Reynolds stress and heat flux in the atmospheric surface layer. *Quart. J. Roy. Meteor. Soc.,* **97,** 168–180.

Hill, R. J., and S. F. Clifford, 1978: Modified spectrum of atmospheric temperature fluctuations and its application to optical propagation. *J. Opt. Soc. Amer.,* **68,** 892–899.

——, and G. R. Ochs, 1992: Surface-layer similarity of the temperature structure parameter. *J. Atmos. Sci.,* **49,** 1348–1353.

Högström, U., 1988: Non-dimensional wind and temperature profiles in the atmospheric surface layer: A re-evaluation. *Bound.-Layer Meteor.,* **42,** 55–78.

——, 1990: Analysis of turbulence structure in the surface layer with a modified similarity formulation for near neutral conditions. *J. Atmos. Sci.,* **47,** 1949–1972.

——, 1996: Review of some basic characteristics of the atmospheric surface layer. *Bound.-Layer Meteor.,* **78,** 215–246.

Holtslag, A. A. M., and H. A. R. De Bruin, 1988: Applied modeling of the nighttime surface energy balance over land. *J. Appl. Meteor.,* **27,** 689–704.

Hsieh, C.-I., G. G. Katul, J. Schieldge, J. Sigmon, and K. R. Knoerr, 1996: Estimation of momentum and heat fluxes using dissipation and flux-variance methods in the unstable surface layer. *Water Resour. Res.,* **32,** 2453–2462.

Jordan, R. E., E. L Andreas, and A. P. Makshtas, 1999: Heat budget of snow-covered sea ice at North Pole 4. *J. Geophys. Res.,* **104,** 7785–7806.

Kaimal, J. C., and J. C. Wyngaard, 1990: The Kansas and Minnesota experiments. *Bound.-Layer Meteor.,* **50,** 31–47.

——, and J. J. Finnigan, 1994: *Atmospheric Boundary Layer Flows: Their Structure and Measurement*. Oxford University Press, 289 pp.

——, J. C. Wyngaard, Y. Izumi, and O. R. Coté, 1972: Spectral characteristics of surface-layer turbulence. *Quart. J. Roy. Meteor. Soc.,* **98,** 563–589.

Kohsiek, W., 1982: Measuring C_T^2, C_Q^2, and C_{TQ} in the unstable surface layer, and relations to the vertical fluxes of heat and moisture. *Bound.-Layer Meteor.,* **24,** 89–107.

Kondo, J., and T. Sato, 1982: The determination of the von Kármán constant. *J. Meteor. Soc. Japan,* **60,** 461–471.

Large, W. G., and S. Pond, 1981: Open ocean momentum flux measurements in moderate to strong winds. *J. Phys. Oceanogr.,* **11,** 324–336.

——, and ——, 1982: Sensible and latent heat flux measurements over the ocean. *J. Phys. Oceanogr.,* **12,** 464–482.

Launiainen, J., and T. Vihma, 1990: Derivation of turbulent surface fluxes—An iterative flux-profile method allowing arbitrary observing heights. *Environ. Software,* **5,** 113–124.

McKeon, B. J., J. Li, W. Jiang, J. F. Morrison, and A. J. Smits, 2004: Further observations on the mean velocity distribution in fully-developed pipe flow. *J. Fluid Mech.,* **501,** 135–147.

Monin, A. S., and A. M. Yaglom, 1971: *Statistical Fluid Mechanics: Mechanics of Turbulence*. Vol. 1. MIT Press, 769 pp.

Nagib, H., C. Christophorou, and P. Monkewitz, 2004: Impact of pressure-gradient conditions on high Reynolds number turbulent boundary layers. *Extended Abstracts, XXI Int. Congress of Theoretical and Applied Mechanics*, Warsaw, Poland, International Union of Theoretical and Applied Mechanics, Paper FM24L_12965.

Obukhov, A. M., 1971: Turbulence in an atmosphere with a non-uniform temperature. *Bound.-Layer Meteor.,* **2,** 7–29.

Oncley, S. P., C. A. Friehe, J. C. LaRue, J. A. Businger, E. C. Itsweire, and S. S. Chang, 1996: Surface-layer fluxes, profiles, and turbulence measurements over uniform terrain under near-neutral conditions. *J. Atmos. Sci.,* **53,** 1029–1044.

Österlund, J. M., A. V. Johansson, H. M. Nagib, and M. H. Hites, 2000: A note on the overlap region in turbulent boundary layers. *Phys. Fluids,* **12,** 1–4.

Panofsky, H. A., 1963: Determination of stress from wind and temperature measurements. *Quart. J. Roy. Meteor. Soc.,* **89,** 85–94.

——, and J. A. Dutton, 1984: *Atmospheric Turbulence: Models and Methods for Engineering Applications*. John Wiley and Sons, 397 pp.

——, H. Tennekes, D. H. Lenschow, and J. C. Wyngaard, 1977: The characteristics of turbulent velocity components in the surface layer under convective conditions. *Bound.-Layer Meteor.,* **11,** 355–361.

Paquin, J. E., and S. Pond, 1971: The determination of the Kolmogoroff constants for velocity, temperature and humidity fluctuations from second- and third-order structure functions. *J. Fluid Mech.,* **50,** 257–269.

Paulson, C. A., 1970: The mathematical representation of wind speed and temperature profiles in the unstable atmospheric surface layer. *J. Appl. Meteor.,* **9,** 857–861.

Perry, A. E., S. Hafez, and M. S. Chong, 2001: A possible reinterpretation of the Princeton superpipe data. *J. Fluid Mech.,* **439,** 395–401.

Resio, D. T., C. E. Long, and C. L. Vincent, 2004: Equilibrium-range constant in wind-generated wave spectra. *J. Geophys. Res.,* **109,** C01018, doi:10.1029/2003JC001788.

Sorbjan, Z., 1989: *Structure of the Atmospheric Boundary Layer*. Prentice Hall, 317 pp.

Taylor, R. J., 1961: A new approach to the measurement of turbulent fluxes in the lower atmosphere. *J. Fluid Mech.,* **10,** 449–458.

Tennekes, H., 1973: The logarithmic wind profile. *J. Atmos. Sci.,* **30,** 234–238.

Webb, E. K., 1970: Profile relationships: The log-linear range, and extension to strong stability. *Quart. J. Roy. Meteor. Soc.,* **96,** 67–90.

Wieringa, J., 1974: Comparison of three methods for determining strong wind stress over Lake Flevo. *Bound.-Layer Meteor.,* **7,** 3–19.

Williams, R. M., and C. A. Paulson, 1977: Microscale temperature and velocity spectra in the atmospheric boundary layer. *J. Fluid Mech.,* **83,** 547–567.

Wyngaard, J. C., and O. R. Coté, 1971: The budgets of turbulent kinetic energy and temperature variance in the atmospheric surface layer. *J. Atmos. Sci.,* **28,** 190–201.

——, and S. F. Clifford, 1978: Estimating momentum, heat and moisture fluxes from structure parameters. *J. Atmos. Sci.,* **35,** 1204–1211.

——, O. R. Coté, and Y. Izumi, 1971a: Local free convection, similarity, and the budgets of shear stress and heat flux. *J. Atmos. Sci.,* **28,** 1171–1182.

——, Y. Izumi, and S. A. Collins Jr., 1971b: Behavior of the refractive-index-structure parameter near the ground. *J. Opt. Soc. Amer.,* **61,** 1646–1650.

——, J. A. Businger, J. C. Kaimal, and S. E. Larsen, 1982: Comments on 'A revaluation of the Kansas mast influence on measurements of stress and cup anemometer overspeeding.' *Bound.-Layer Meteor.,* **22,** 245–250.

Yaglom, A. M., 1977: Comments on wind and temperature flux-profile relationships. *Bound.-Layer Meteor.,* **11,** 89–102.

Yelland, M. J., and P. K. Taylor, 1996: Wind stress measurements from the open ocean. *J. Phys. Oceanogr.,* **26,** 541–558.

——, B. I. Moat, P. K. Taylor, R. W. Pascal, J. Hutchings, and V. C. Cornell, 1998: Wind stress measurements from the open ocean corrected for airflow distortion by the ship. *J. Phys. Oceanogr.,* **28,** 1511–1526.

Zagarola, M. V., and A. J. Smits, 1998: Mean-flow scaling of turbulent pipe flow. *J. Fluid Mech.,* **373,** 33–79.

Zanoun, E.-S., F. Durst, and H. Nagib, 2003: Evaluating the law of the wall in two-dimensional fully developed turbulent channel flows. *Phys. Fluids,* **15,** 3079–3089.