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On the Role of Wave Transience and Dissipation in Stratospheric Mean Flow Vacillations¹

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ABSTRACT

The quasi-geostrophic β -plane channel model of Holton and Mass (1976) is used to further elucidate the nature of mean flow changes in the stratosphere forced by planetary waves generated in the troposphere. It is shown that wave transience, not dissipation, is the primary mechanism for generating mean flow oscillations. In addition it is shown that the critical wave-forcing amplitude necessary to produce a vacillating response is very sensitive to the initial mean flow profile.

1. Introduction

Recently, Holton and Mass (1976)² used a quasi-geostrophic β -plane channel model to study the stratospheric response to vertically propagating planetary waves excited by steady tropospheric forcing. They found that when the forcing exceeded a certain critical amplitude “vacillation cycles” occurred in which the zonal mean flow and eddy heat fluxes oscillated quasi-periodically. The purpose of this note is to further elucidate the nature of these vacillation cycles in the context of recent theoretical results on the dynamics of wave-mean flow interaction (Andrews and McIntyre, 1976; Boyd, 1976).

Dickinson (1969) has discussed the dynamics of planetary wave-mean flow interactions for quasi-geostrophic motions. The essential requirement for nonzero mean-flow forcing by the waves is that the waves transport quasi-geostrophic potential vorticity meridionally. Here we present the quasi-geostrophic equivalent of the formula (5.5a) of Andrews and McIntyre (1976) in which this transport of potential vorticity is explicitly described in terms of wave transience (proportional to the time rate of change of wave amplitude) and dissipation (proportional to thermal damping of the wave). We show here that

wave transience is indeed the dominant form of mean-flow forcing by the waves for *both* the “vacillating” and “non-vacillating” regimes of HM, although the vacillating case is characterized by a factor of 10 increase in the *relative* importance of dissipation with respect to transience. Furthermore, mean flow forcing by wave transience itself is greatly enhanced in the vacillating case above that of the non-vacillating case, occasionally by as much as a factor of 100.

In addition to these findings, the HM model has been found to be extremely sensitive to the choice of initial zonal wind profile. A brief discussion of this dependence is presented in Section 2; Section 3 deals with the aspects of potential vorticity transport discussed above. The reader is referred to HM for a detailed discussion of the numerical model, and definition of all symbols.

2. Dependence of vacillation on zonal wind profile

For consistency, the model initial and boundary conditions have been chosen as follows:

$$U_0(t=0) = U_R,$$

$$U_0(z=0) = U_R(z=0),$$

$$\frac{\partial U_0}{\partial z}(z=z_T) = \frac{\partial U_R}{\partial z}(z=z_T),$$

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² Hereafter referred to as HM.

where U_0 is the zonal wind amplitude defined by

$$\bar{u} = U_0 \sin ly$$

and U_R is the "radiative equilibrium" profile defined by

$$\bar{u}_R = U_R \sin ly.$$

Thus, the initial mean flow profile is identical to the "radiative equilibrium" profile, and the mean flow at the lower boundary is held constant in time. For simplicity a linear initial profile is chosen, i.e.,

$$U_R = U_B + \Lambda z.$$

In HM only a single initial mean wind profile was investigated. We have found, however, that the onset of vacillation cycles is extremely sensitive to the values of U_B and Λ . Table 1 shows the value of critical forcing (the minimum wave amplitude forcing required to produce vacillation) for various values of U_B and Λ . Shown in the same table is the turning point of the steady-state refractive index n , where

$$n^2 = \frac{\beta_e' N^2}{\epsilon U_R f_0^2} - \frac{N^2}{f_0^2} (k^2 + l^2) - \frac{1}{4H^2}.$$

(The contribution to n from Newtonian cooling is negligible since we are sufficiently far from zero-wind lines here.) The turning point is the height at which $n=0$, and where, in the sense of steady-state theory, the waves become "external" in the vertical.

It is apparent from Table 1 that the critical forcings are generally compatible with the expectations of steady-state vertical wave propagation. Thus, waves which can propagate to higher levels have smaller values of the critical forcing. Nevertheless, the critical forcings seem surprisingly sensitive to U_R . As implied in the Introduction, we have indeed found that the magnitude of forcing by wave transience, and not merely the depth of steady-state vertical wave propagation, is a determining factor in the onset of vacillation; this point is discussed in the next section.

3. Potential vorticity transport and wave-zonal flow interaction

a. Theory

In the quasi-geostrophic model the necessary criterion for mean-flow forcing by the waves is simply that the waves transport potential vorticity in the meridional direction (Dickinson, 1969). Since the HM model is governed by conservation of quasi-geostrophic potential vorticity, the wave and mean-flow equations become, respectively,

$$(\partial/\partial t + ik\epsilon U_0)q_0 + \beta_e' ik\Psi + D_0 = 0, \quad (3.1)$$

$$\frac{1}{\epsilon} \frac{\partial \beta_e'}{\partial t} = -\bar{D} + \frac{1}{2} l^2 \epsilon k \exp(z/H) \text{Im}(\Psi^* q_0), \quad (3.2)$$

TABLE 1. Critical forcing amplitude (uncertainty shown in parentheses) for various values of U_B and Λ . Units: m. Also shown is the turning point height for the refractive index n (shown bold face). Units: km.

Λ (m s ⁻¹ km ⁻¹)	U_B (m s ⁻¹)			
	0	5	10	15
1			33(17) 22.1	110(10) 17.1
2		<50 18.3	130(10) 15.8	308(1) 13.3
3	<50 17.1	90(10) 15.4	270(10) 13.7	550(50) 12.1

Critical forcing (uncertainty) (m).
Turning point (km).

where

$$q_0 = \left[- (k^2 + l^2) + \frac{f_0^2}{N^2} \left(\frac{\partial^2}{\partial z^2} - \frac{1}{4H^2} \right) \right] \Psi, \quad (3.3)$$

$$\beta_e' = \beta + \epsilon l^2 U_0 - \frac{f_0^2}{N^2} \left(\frac{\partial^2 U_0}{\partial z^2} - \frac{1}{H} \frac{\partial U_0}{\partial z} \right), \quad (3.4)$$

$$D_0 = \frac{f_0^2}{N^2} \left(\frac{\partial}{\partial z} - \frac{1}{2H} \right) \left[\alpha \left(\frac{\partial}{\partial z} + \frac{1}{2H} \right) \Psi \right], \quad (3.5)$$

$$\bar{D} = - \frac{f_0^2}{N^2} \left(\frac{\partial}{\partial z} - \frac{1}{H} \right) \left[\alpha \left(\frac{\partial U_0}{\partial z} - \frac{\partial U_R}{\partial z} \right) \right]. \quad (3.6)$$

[Recall that

$$\Psi' = \text{Re}[\Psi(z,t) \exp ikx] \exp(z/2H) \sin ly \quad (3.7)$$

and thus subsequent discussion assumes a single harmonic in both zonal and meridional directions.] Multiplying (3.1) by q_0^* and the conjugate of (3.1) by q_0 and summing the resulting equations we obtain

$$k \text{Im}(\Psi^* q_0) = -\beta_e'^{-1} [\text{Re}(D_0 q_0^*) + \frac{1}{2} \partial |q_0|^2 / \partial t], \quad (3.8)$$

in which, via (3.2), mean flow forcing by the waves is expressed in terms of wave transience and dissipation.³

b. Model results

Model parameters have been chosen such that $U_B = 15$ m s⁻¹ and $\Lambda = 2$ m s⁻¹ km⁻¹; the Newtonian

³ We remark that an interesting corollary of (3.2) and (3.8) is obtained in the absence of wave and zonal mean dissipation; viz.,

$$[\beta_e'(t)]^2 - [\beta_e'(0)]^2 = -\frac{1}{2} \epsilon^2 l^2 \exp(z/H) [|q_0(t)|^2 - |q_0(0)|^2].$$

Thus if $|q_0(t)| = |q_0(0)|$, it follows that $\beta_e'(t) = \beta_e'(0)$; that is to say, wave transience alone cannot give rise to permanent changes in the mean flow. We have not realized such a result in our numerical model, however, because of the apparent excitation of a permanent free mode by the initial lower boundary forcing.

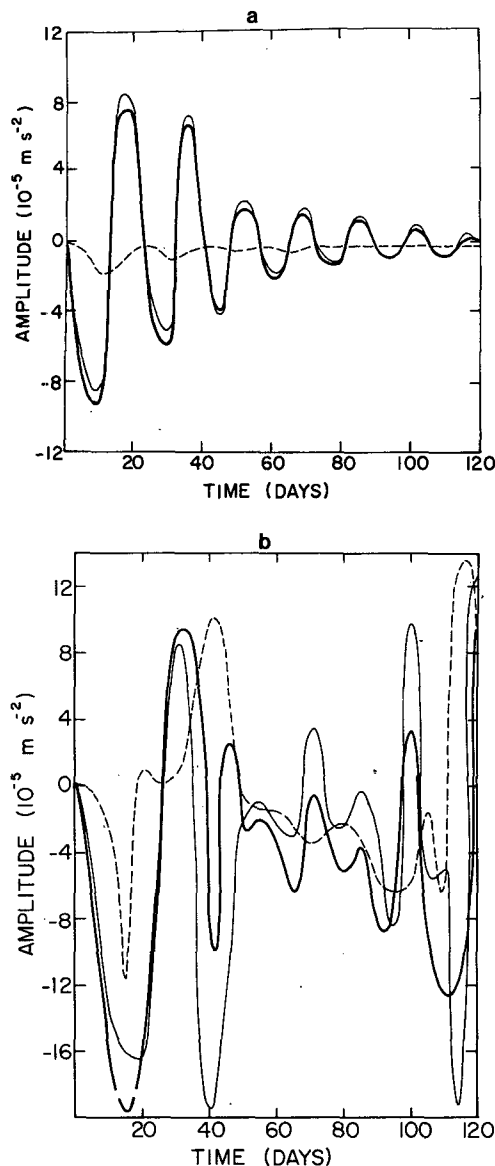


FIG. 1. Potential vorticity transport [see Eq. (3.8)] and its contributions from wave transience and dissipation for (a) non-vacillating case, $h_B = 300$ m and (b) vacillating case, $h_B = 320$ m. Units: 10^{-5} m s^{-2} . Thick solid line, total transport; thin solid line, transience; dashed line, dissipation. (Note that total transport is off the scale near day 18.)

cooling profile is identical to that of HM. In all cases discussed here we confine attention to wavenumber 2.

Figs. 1a and 1b show the potential vorticity transport and its contributions from wave transience and dissipation for $z = 20$ km, for $h_B = 300$ m and 320 m, respectively. Note that, according to Table 1, these two forcings produce non-vacillating and vacillating cases, respectively. From these figures we observe the following:

1) The first 15 days of the model run (which might be labeled the "pre-warming" stage, since the initial development of the model flow is essentially that of

a sudden warming) show that wave transience is an order of magnitude larger than wave dissipation in both cases.

2) In the non-vacillating case after day 15 wave transience continues to dominate wave dissipation, although both are decreasing in magnitude as the waves approach stationarity.

3) In the vacillating case, on the other hand, wave dissipation and transience become of comparable magnitude after day 15. Close inspection of Fig. 1b, nevertheless, reveals that the essential features of the total potential vorticity transport are closely paralleled by wave transience alone. This suggests that the vacillations do not depend on wave dissipation for their existence. We have established this with a model run (not shown) in which wave dissipation was set to zero; similar vacillation cycles were observed, although the vacillations began almost a week earlier. The latter observation is consistent with Geisler's (1974) finding that Newtonian cooling acted to delay the onset of a sudden warming.

In comparing the two flow regimes, the question has arisen as to whether or not a continuous transition (as a function of forcing) exists between them. Previous runs seemed to suggest the contrary: for example, Fig. 2 of HM shows an apparent discontinuity in the behavior of the zonal wind at about 20 days (which is 10 days *before* the onset of a zero wind line at that level). To answer this question two time-height cross sections of zonal winds are presented in Figs. 2a and 2b, with forcing of $h_B = 307$ and 308 m, respectively (non-vacillating and vacillating cases, respectively). Although zonal winds begin to diverge slightly in behavior after day 16, there is indeed a strong "discontinuity" in model behavior at day 22, with the rapid onset of easterlies in the vacillating regime.

This discontinuity is suggestive of the "suddenness" of sudden warmings. We have found, however, that this discontinuity has its origin in the behavior of wave transience around day 14 and thereafter. Figs. 3a and 3b show time-height cross sections of wave transience for the same time periods as in Fig. 2. In the non-vacillating case the downgradient (negative) values of potential vorticity transport are replaced after day 14 by a descending region of upgradient transport, which acts to reestablish the strong westerlies characteristic of this regime. In contrast, in the vacillating case this region of upgradient transport is unable to descend to all levels of the model, and is quickly replaced by a continuation of the downgradient transport, leading to the vacillation cycles (sudden warming and subsequent warmings).⁴

⁴ Presumably the observed differences in wave transience in the two cases are due to slight differences in wave propagation at these times. It is remarkable, nevertheless, that these "slight differences" are so greatly amplified as (in the vacillating case) the zonal wind and the Doppler-shifted wave frequency go to zero.

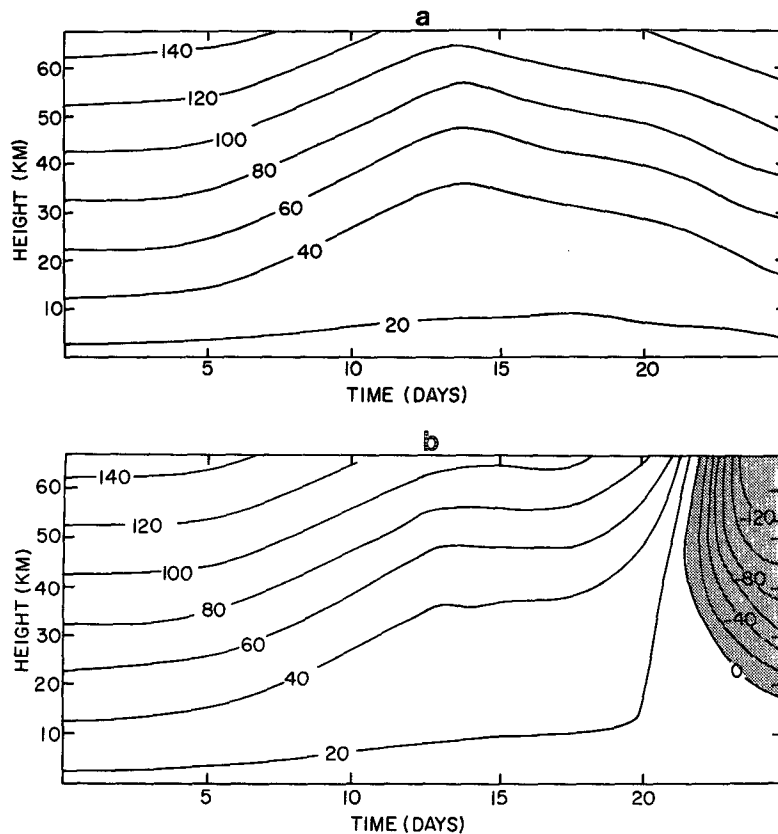


FIG. 2. Time-height cross sections of zonal wind for model runs: (a) non-vacillating case, $h_B = 307$ m and (b) vacillating case, $h_B = 308$ m. Units: m s^{-1} .

It was remarked under comment 3) above that the vacillating case is characterized by greatly enhanced potential vorticity transport over that of the non-vacillating case. We have found that these large transports are generally associated with the onset of zero wind lines in the model. This result is consistent with physical intuition: for stationary-phase speed waves, a zero wind line would allow algebraic, monotonically increasing particle displacements, giving rise to downgradient potential vorticity "dispersion" (Dickinson, 1969) linearly dependent on time. Of course, in a time-dependent model as that of HM, such "critical levels" are present at a given height for *finite* times, on the order of a day or so; hence this dispersion may, or may not, become of large finite amplitude (but *not* of infinite amplitude suggested by a discontinuity in the vertical gradient of eddy heat flux as is found by "steady-state" reasoning!) In this sense, the HM model clarifies the role of transient critical levels in the atmosphere, particularly in regard to sudden warmings.

4. Conclusion

The behavior of the "vacillation cycles" model of HM is in general agreement with the predictions of steady-state vertical wave propagation as discussed

by Charney and Drazin (1961) and Simmons (1974) in the sense that waves which are able to propagate to large heights according to the steady-state theory can produce vacillations at weaker amplitudes than can those waves which have turning points at low levels. Nevertheless, an important feature is found which is unique to time-dependent models, namely, the role of wave transience in wave-mean-flow interactions. Such transience is undoubtedly important in accounting for the sensitivity of the onset of vacillation in the model to the mean flow profiles and wave amplitude forcings. We suggest that this sensitivity may be an important factor in controlling the occurrence of sudden warmings in the atmosphere. This idea is consistent with the absence of major warmings in the Southern Hemisphere, where wave amplitudes are weak and zonal winds strong⁵ in comparison to those of the Northern Hemisphere (where major warmings are observed).

Although the present model is a highly simplified representation of the stratosphere, it contains enough of the essential dynamics so that we may conclude

⁵ Thus, instability appears an unlikely cause of the warmings; if anything, the polar night jet of the Southern Hemisphere appears to be more unstable (Leovy and Webster, 1976) than that of the Northern Hemisphere.

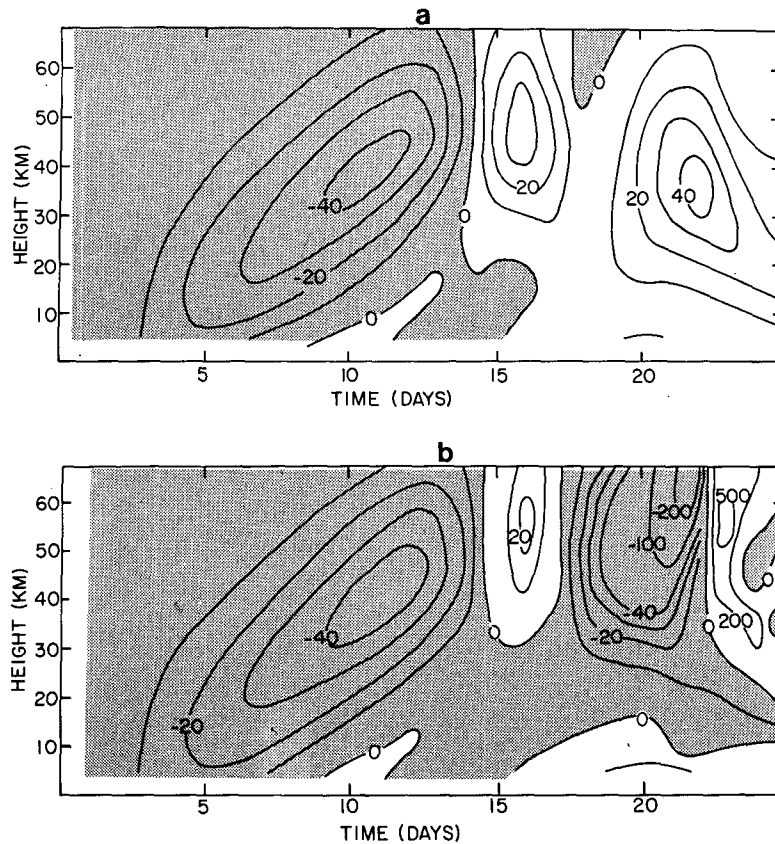


FIG. 3. Time-height cross sections of the wave transience, $\frac{1}{2} \partial |q_0|^2 / \partial t$, for the nonvacillating case (a) and vacillating case (b) shown in Fig. 2. Units: s^{-3} . (Note the nonuniform contour interval in 3b.)

with confidence that in attempting to model atmospheric behavior for time scales shorter than seasonal, a time-dependent model is imperative. This is particularly true if critical levels are expected to be present. Furthermore, one must exercise a great deal of caution in attempting to extend ideas developed for steady-state models to the real atmosphere in which such critical levels may be highly transient.

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